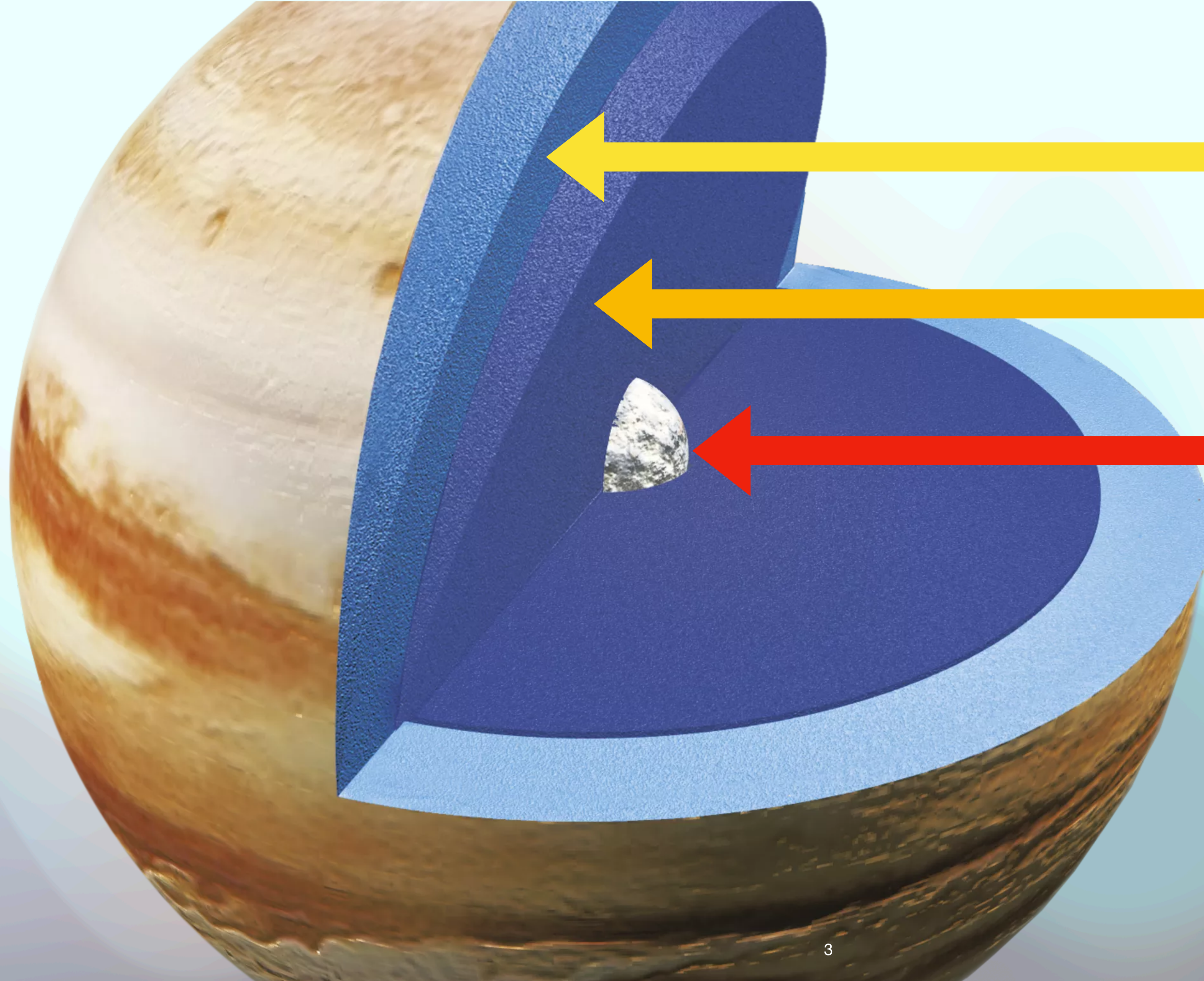


On theoretical aspects of atmospheres and interiors

**PhD supervised by
B. Charnay, AM. Lagrange, S. Mazevet**

Christian Wilkinson - 07.09.2022





Atmosphere :
Interacts with
surrounding
 $\sim 1e2-1e3$ K
 ~ 1 bar

Interior : Linked to
the atmospheric
dynamics/
composition
 $\sim 1e3-1e4$ K
 $\sim 1-1e7$ bar

Core :
Composed of
heavier elements,
layout unknown
 $\sim 1e4$ K
 $\sim 1e11$ Pa



ultraviolet

visible light

infrared

317.5 nm

325 nm

340 nm

388 nm

443 nm

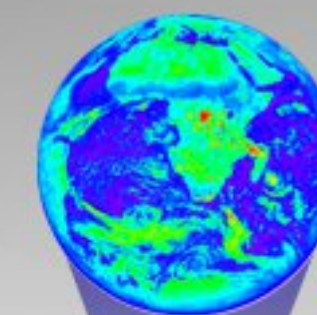
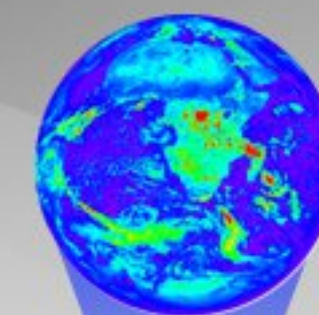
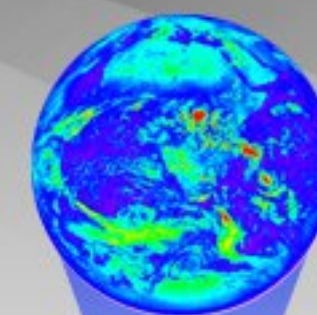
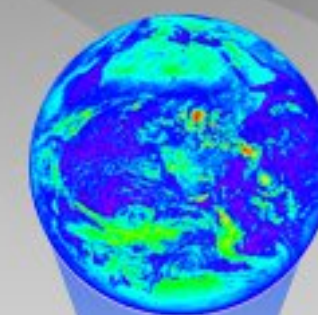
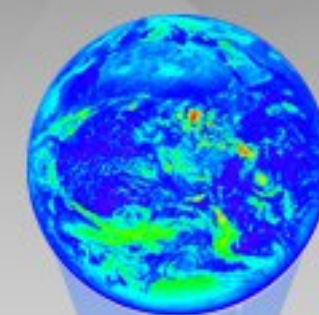
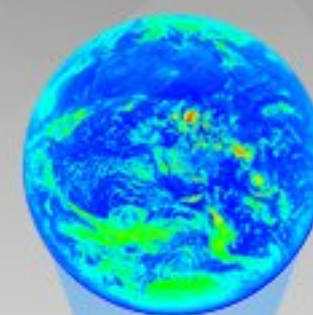
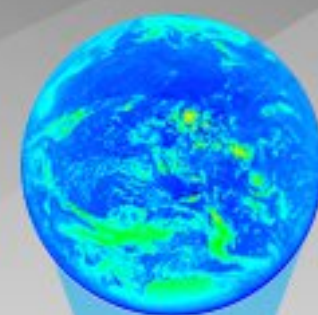
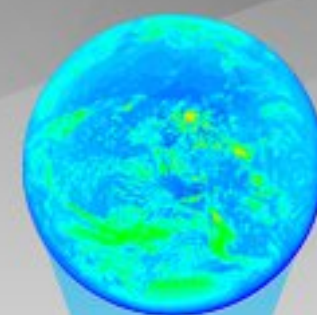
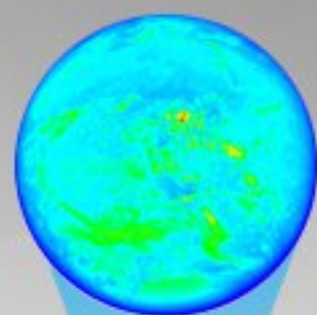
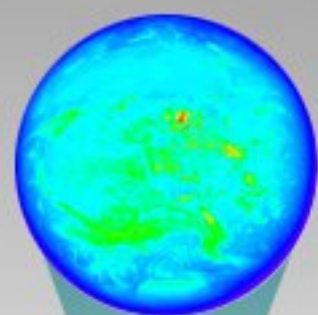
551 nm

680 nm

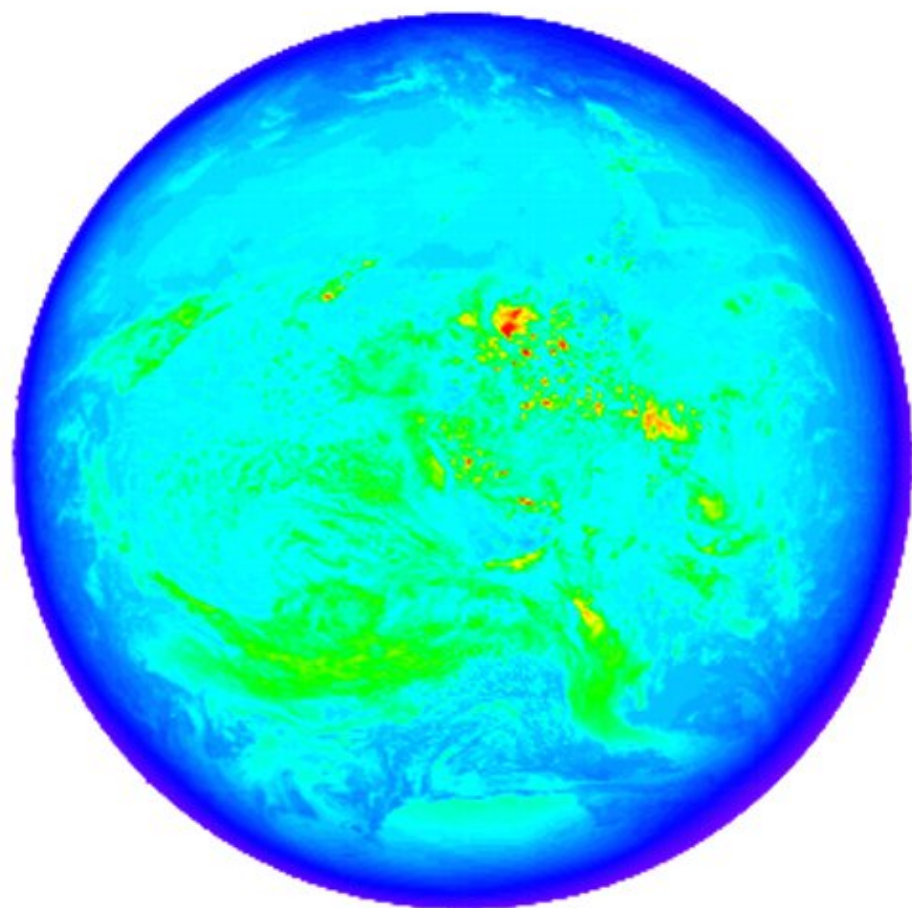
688 nm

764 nm

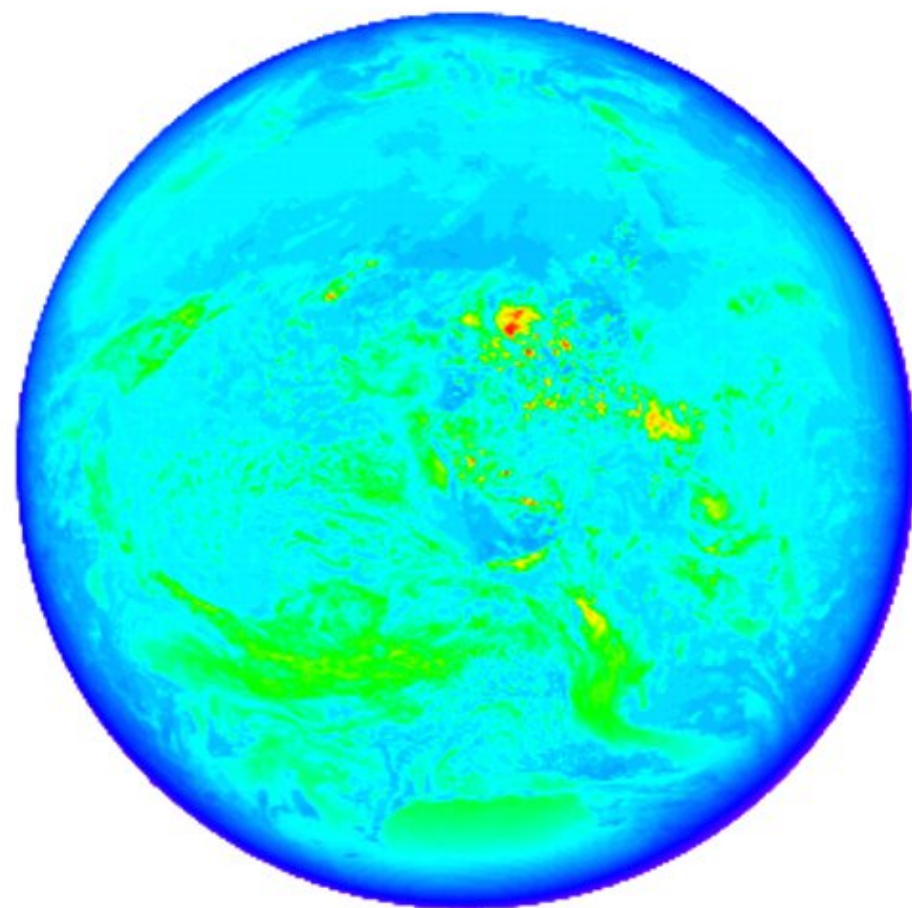
779.5 nm



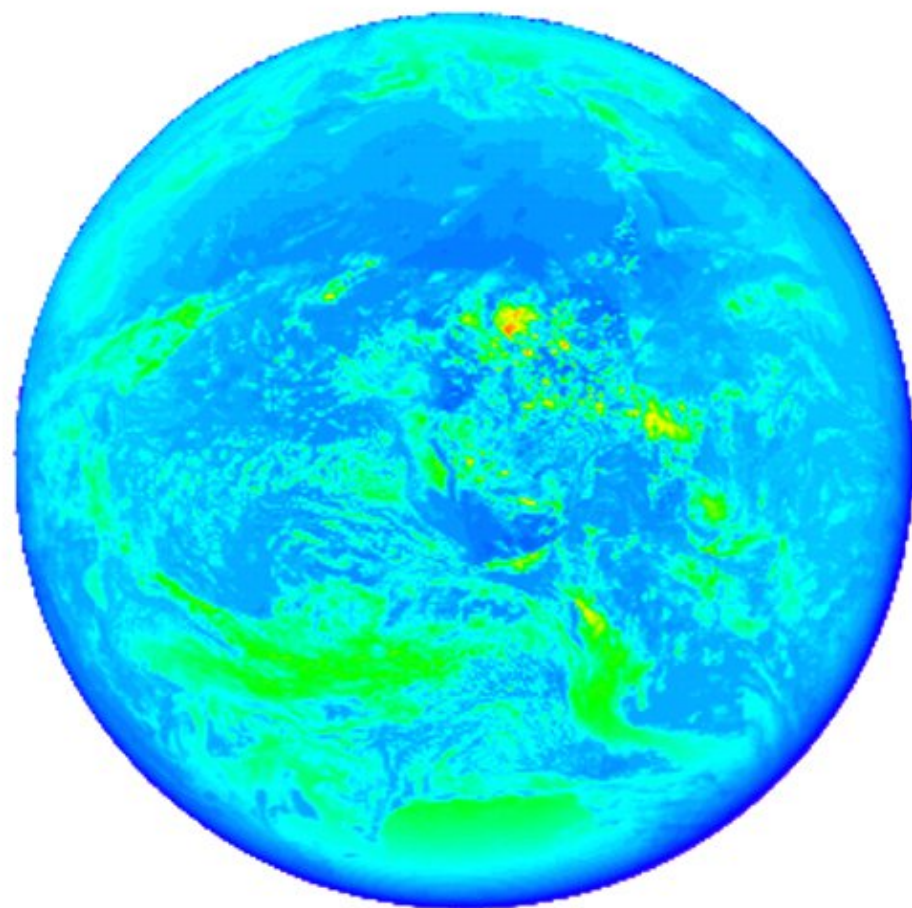
317.5 nm



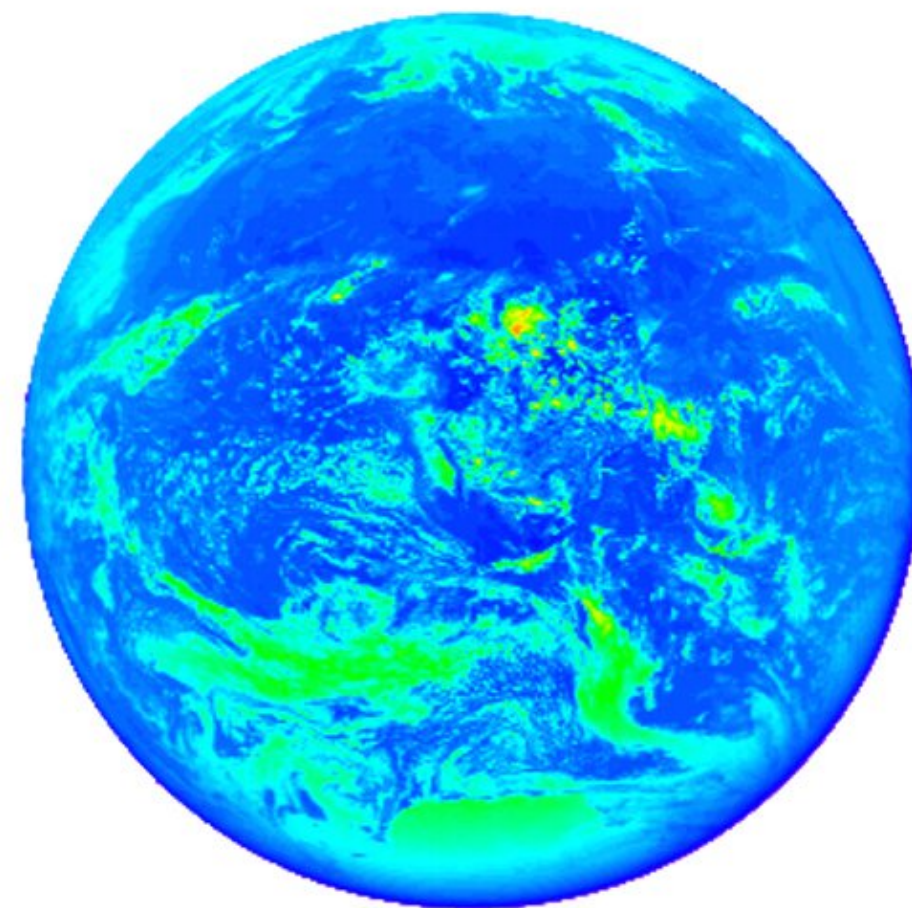
325 nm



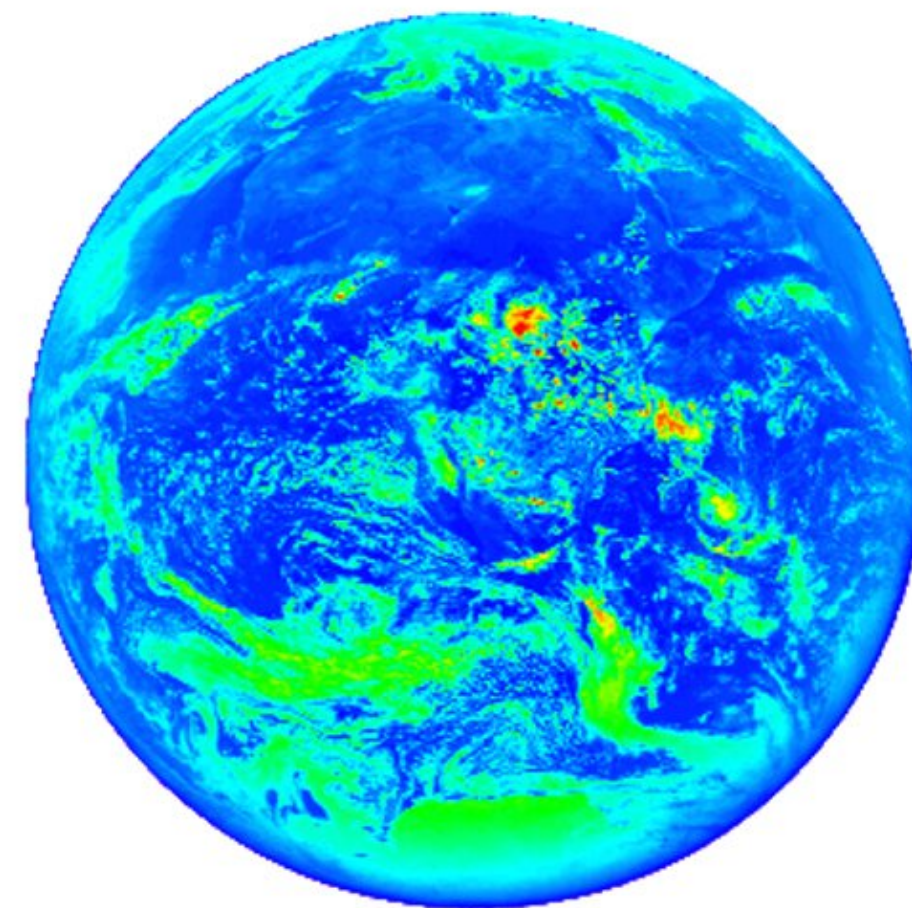
340 nm



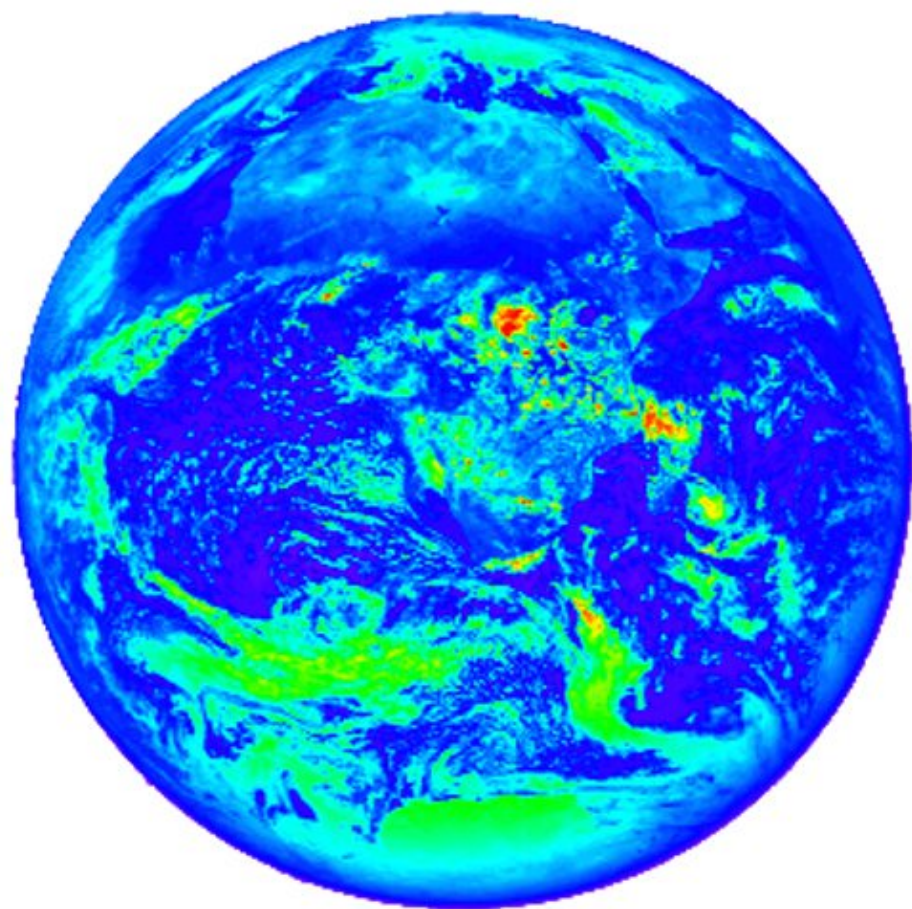
388 nm



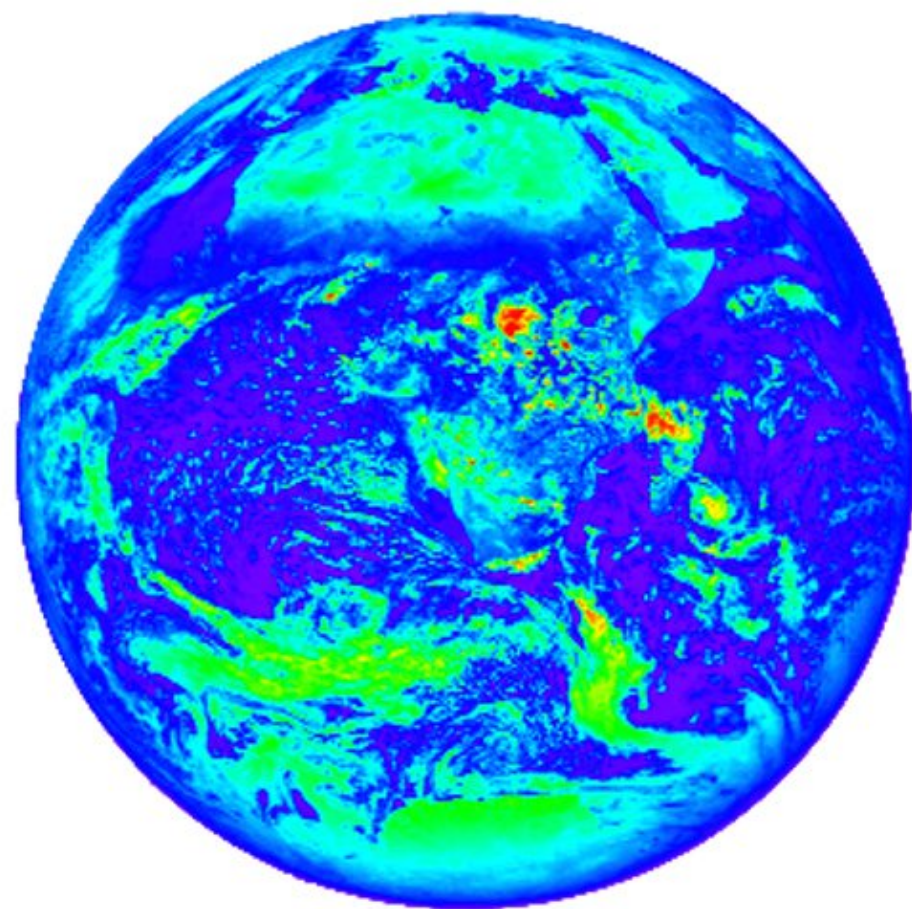
443 nm



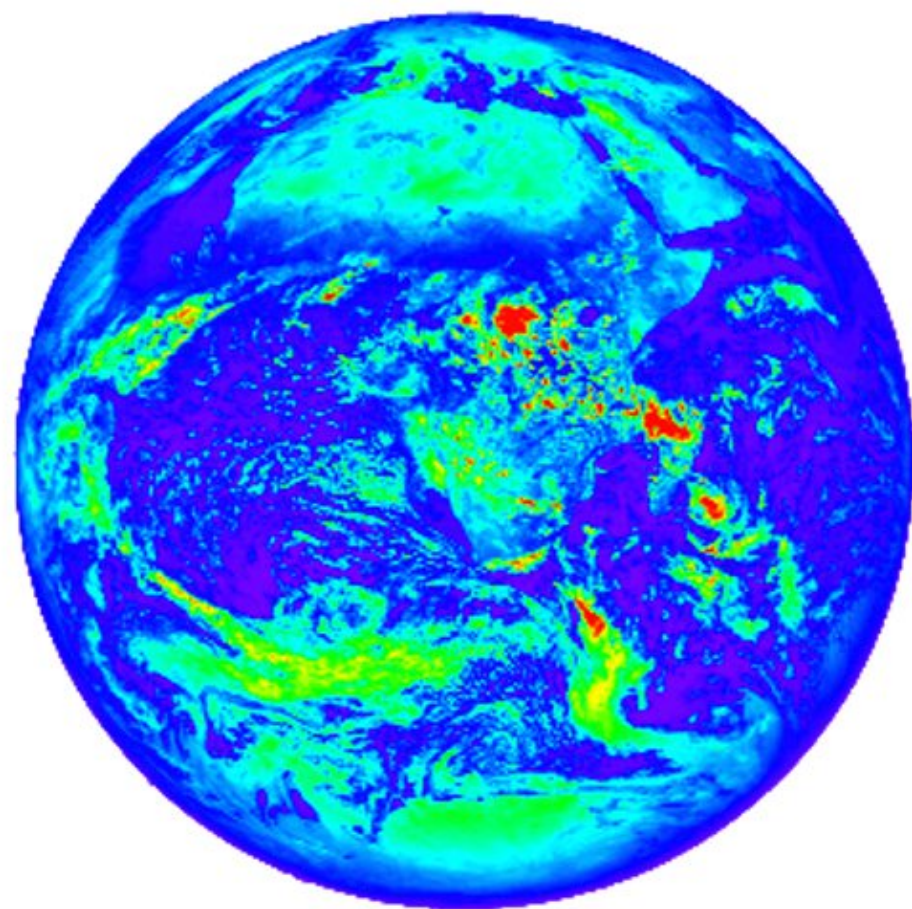
551 nm



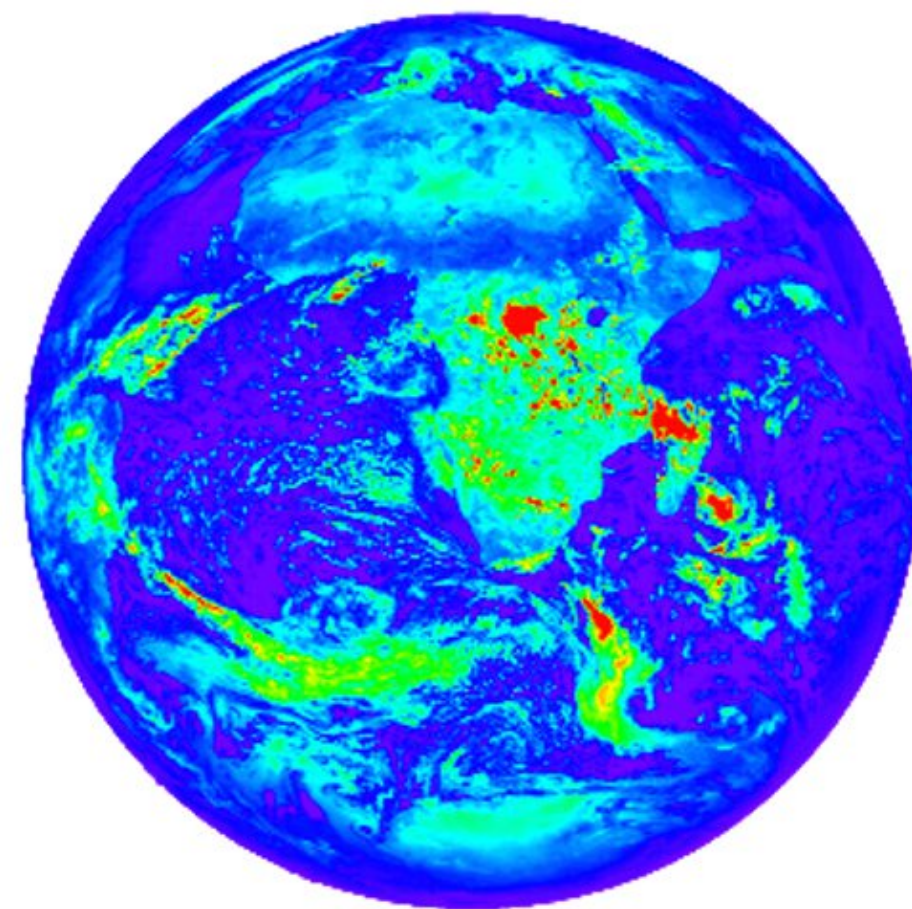
680 nm



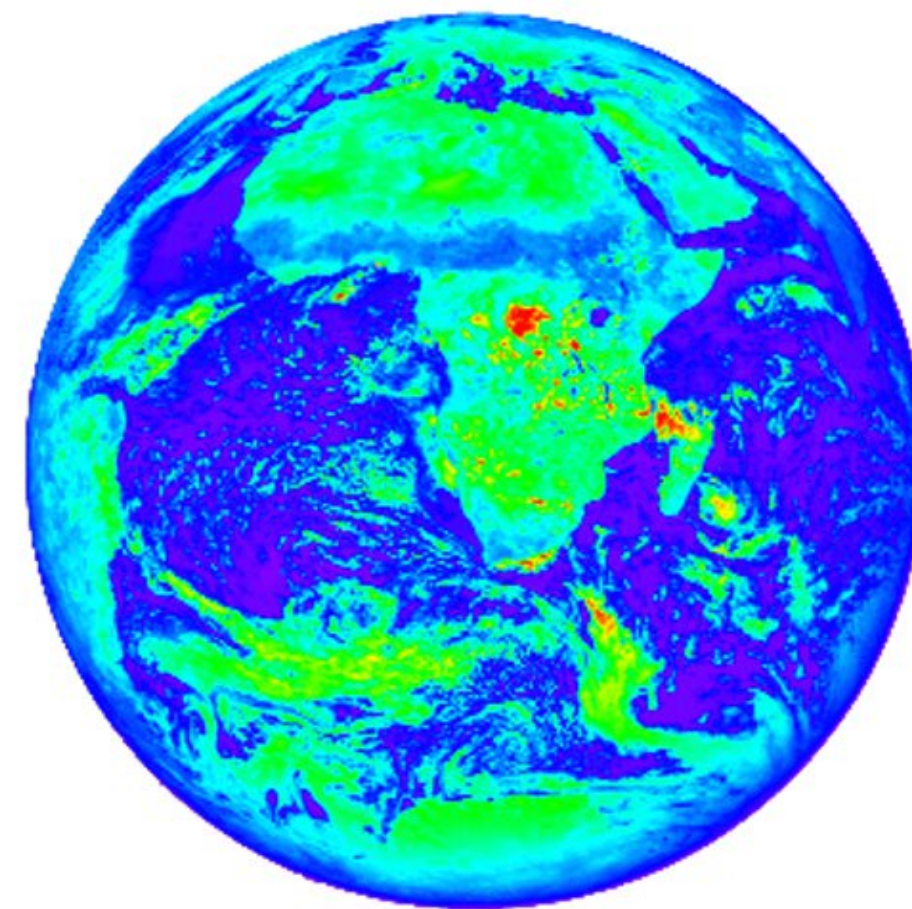
688 nm

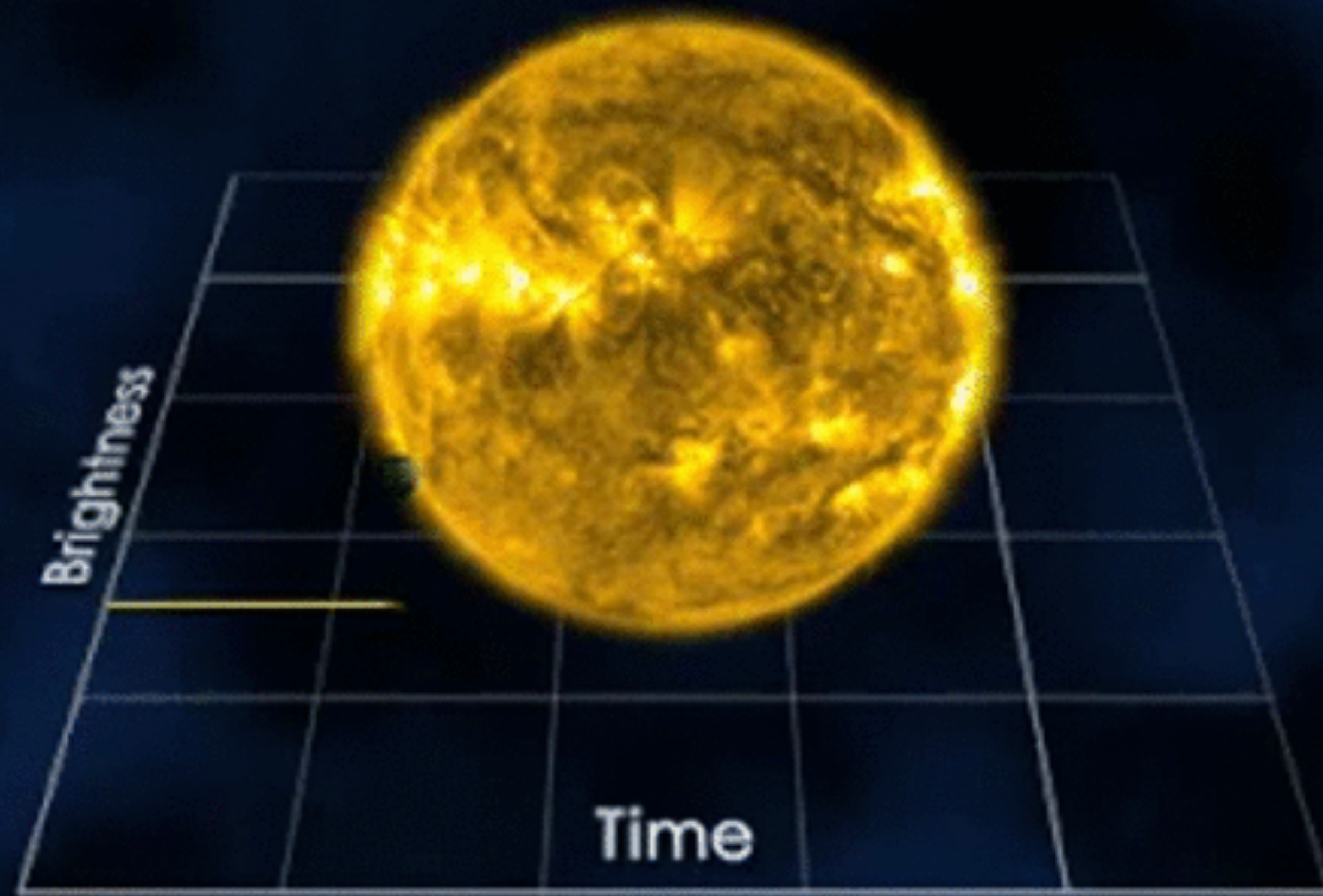


764 nm



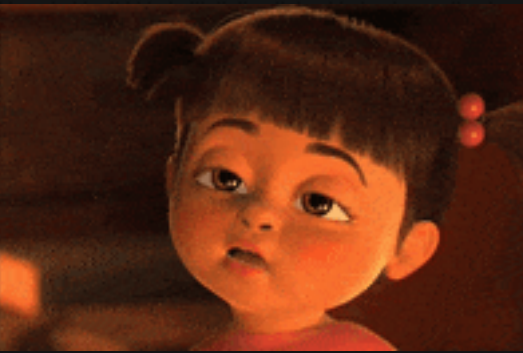
779.5 nm





$$\delta_{tra} = \left(\frac{R_p}{R_*} \right)^2$$

Atmospheric scale height



$$\frac{dP}{dz} = -g\rho$$

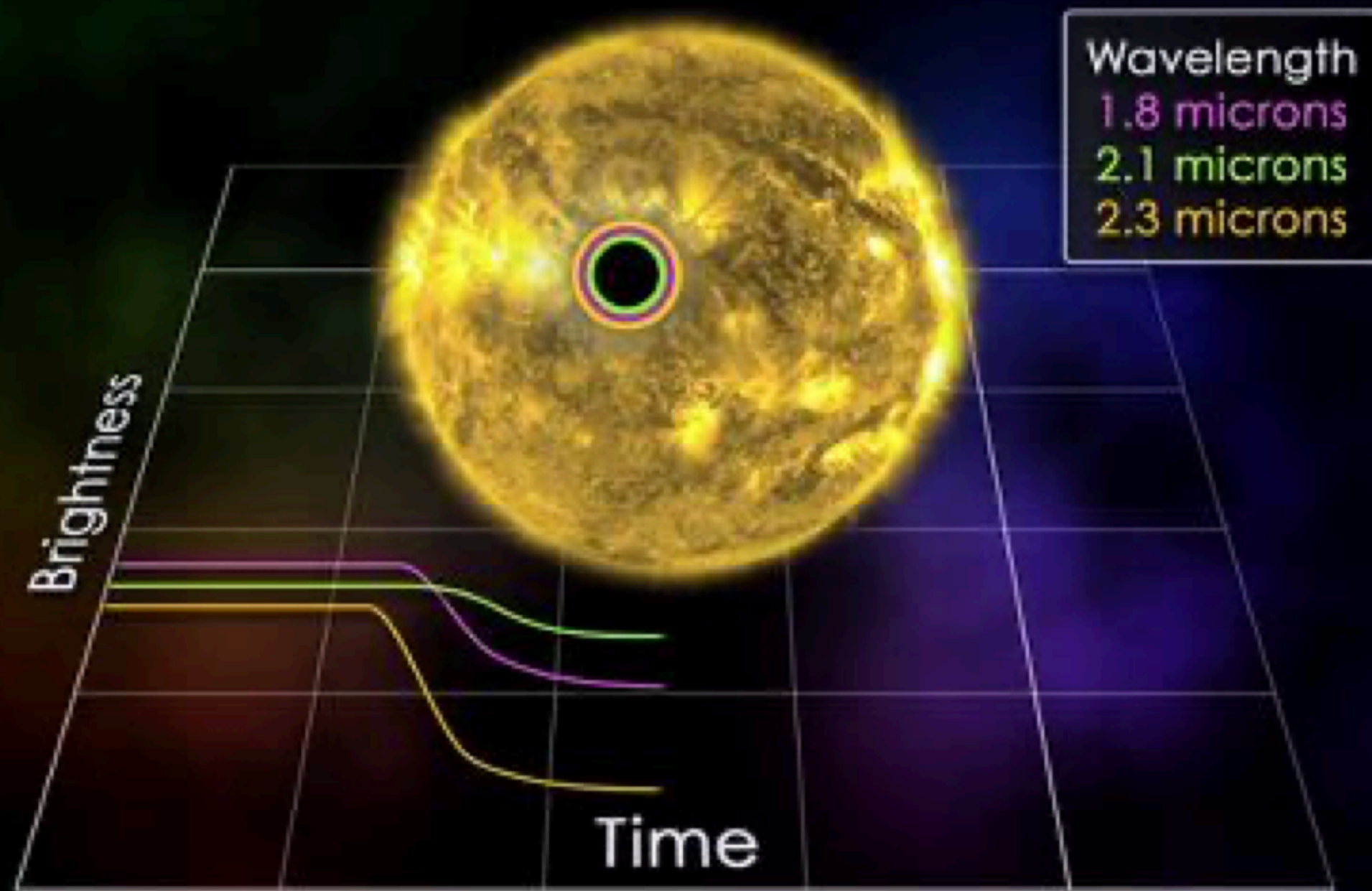
$$PV = NK_bT = \frac{M}{\mu m_h} k_b T \quad P = \frac{\rho}{\mu m_h} k_b T$$

$$\int_{P_0}^P \frac{dP}{P} = \int_z^0 -\frac{\mu m_h g}{k_b T} dz$$

$$P = P_0 e^{-\frac{\mu m_h g}{k_b T} z} = P_0 e^{-\frac{z}{H}}$$

$$H = \frac{k_b T}{\mu m_h g}$$

Variation of transit depth



$$\Delta\delta_{tra} = \frac{\pi(R_p + N_H H)^2}{\pi R_\star^2} - \frac{\pi R_p^2}{\pi R_\star^2} \approx 2N_H \delta_{tra} \left(\frac{H}{R_p} \right)$$

For an Sun-like star:

- Hot Jupiter ($T=1300$ K, $g=25$ m s⁻², $M=2.3$ g/mol): $\delta_{tra} \approx 0.01$, $\Delta\delta_{tra} \approx 4 \cdot 10^{-4}$
- Earth-like planet ($T=280$ K, $g=10$ m s⁻², $M=28$ g/mol): $\delta_{tra} \approx 10^{-4}$, $\Delta\delta_{tra} \approx 2 \cdot 10^{-6}$

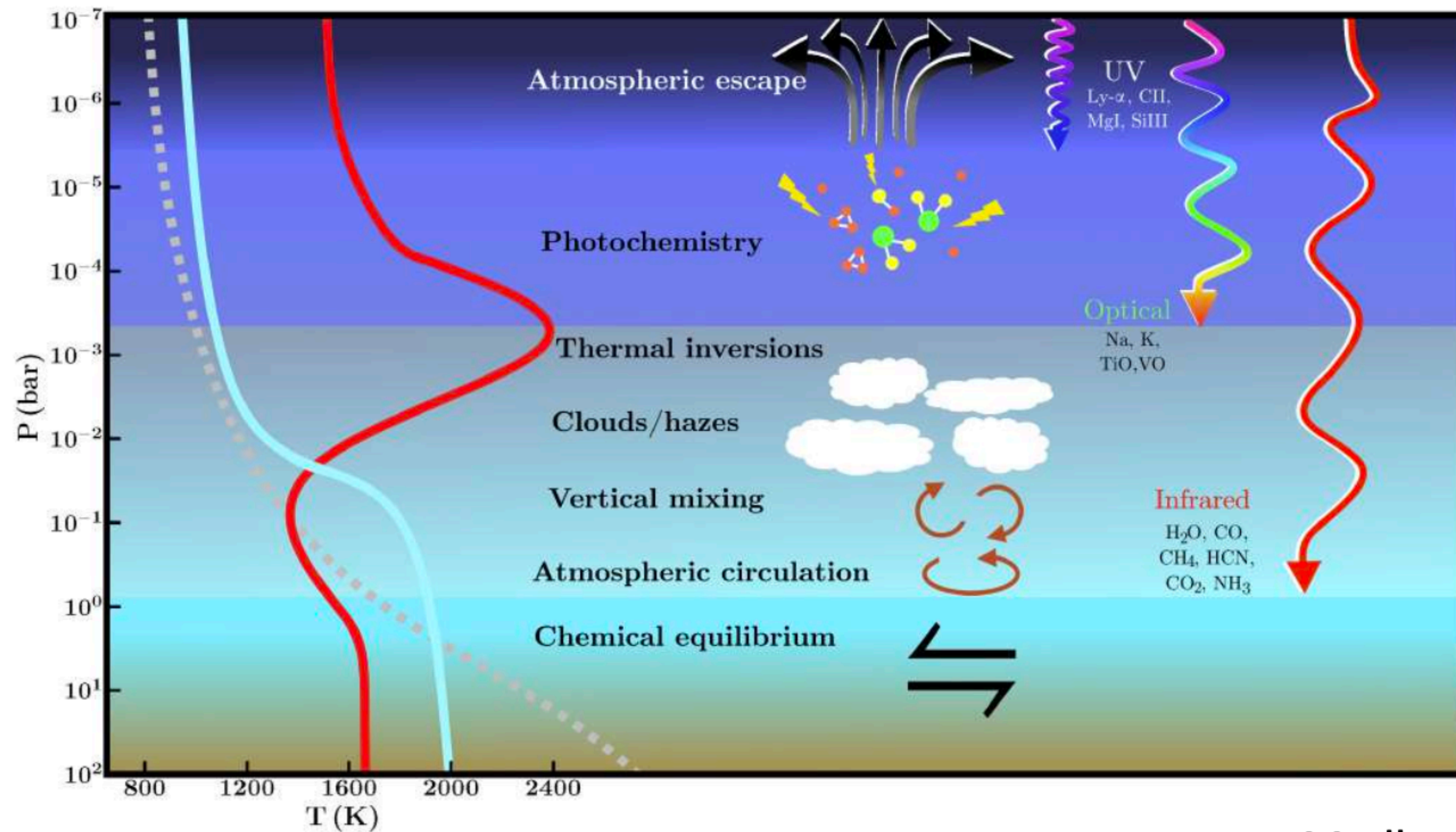
Summary

From the atmospheres to the interior

- Atmospheres :
 - Structure & Dynamics : Modeling
- Interiors :
 - Structure & Dynamics : Modeling
- Linking the two :
 - Complete structural modeling

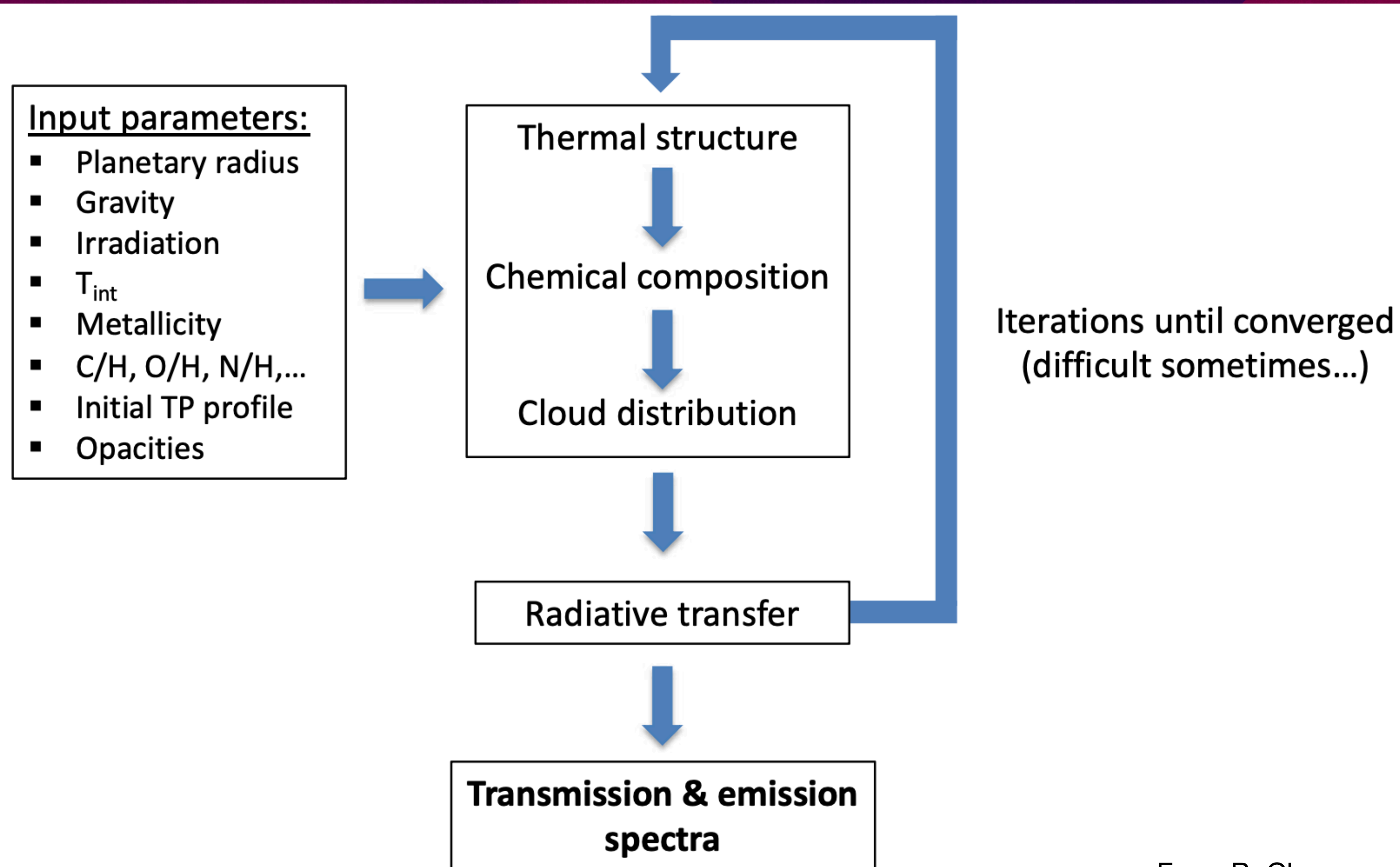
Atmospheres

Summary



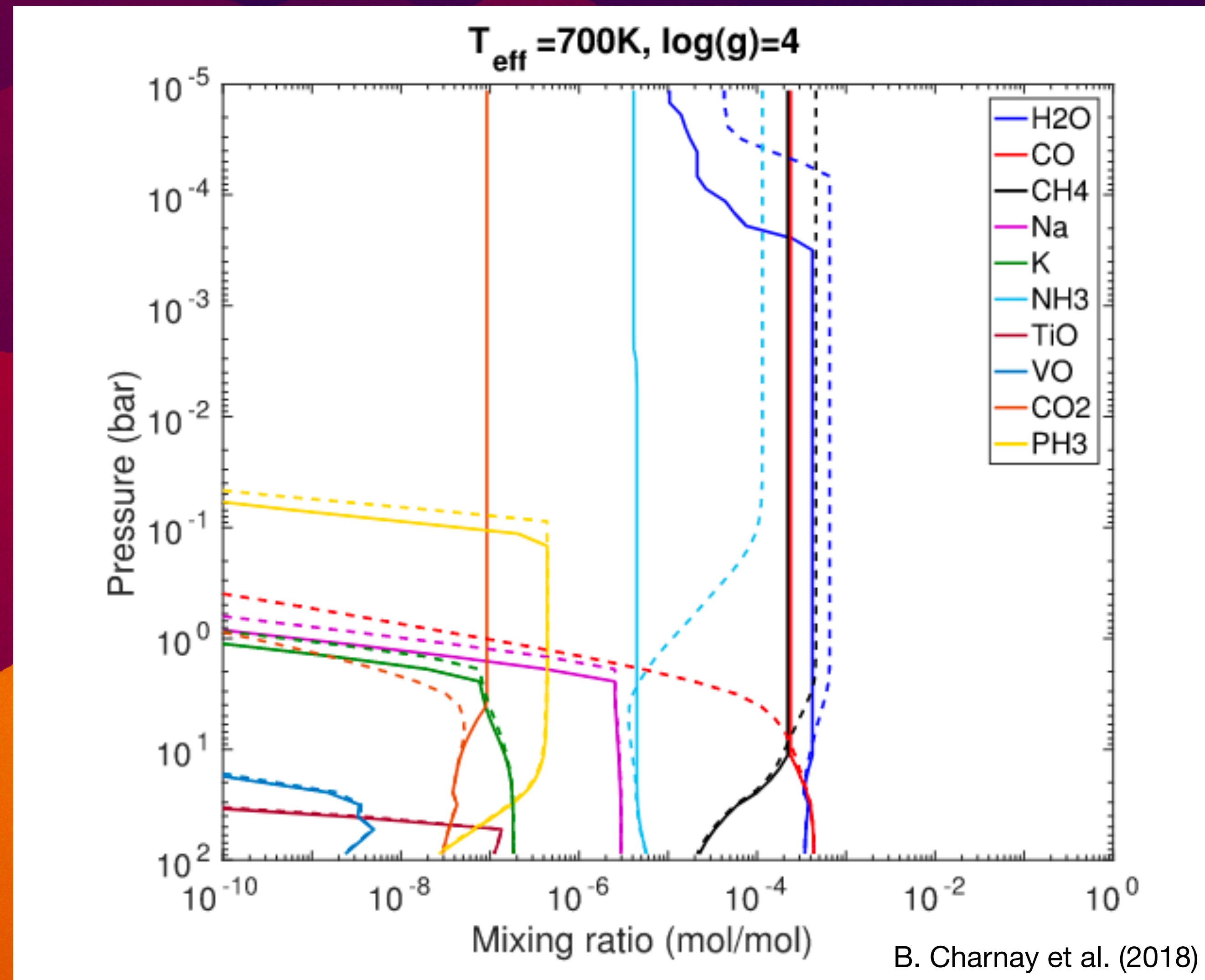
Madhusudhan 2019

Self-consistent atmosphere models



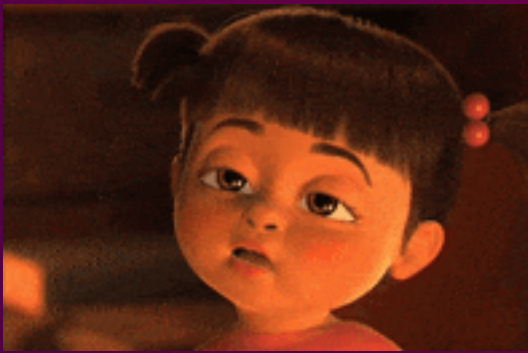
From B. Charnay

Chemical composition



Equilibrium Chemistry

Using thermodynamics



$$\Delta \ln(n_j) - \sum_{i=1}^l a_{ij} \pi_i - \Delta \ln(n) = -\frac{\mu_j}{RT} \quad j = (1, \dots, NG)$$

$$-\sum_{i=1}^l a_{ij} \pi_i = -\frac{\mu_j}{RT} \quad j = (NG + 1, \dots, NS)$$

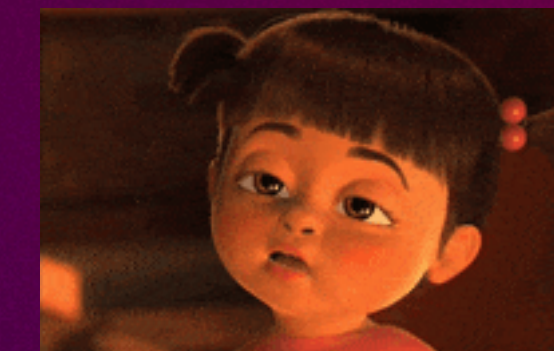
$$\sum_{j=1}^{NG} a_{kj} n_j \Delta \ln(n_j) - \sum_{j=NG+1}^{NS} a_{kj} \Delta n_j = b_k^o - \sum_j a_{kj} n_j$$

$$\sum_{j=1}^{NG} n_j \Delta \ln(n_j) - n \Delta \ln(n) = n - \sum_{j=1}^{NG} n_j$$

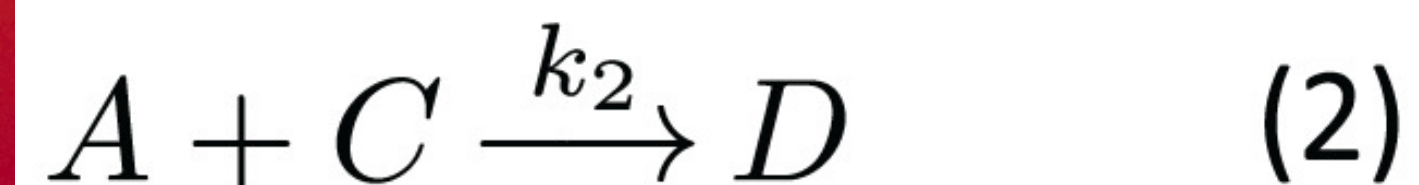
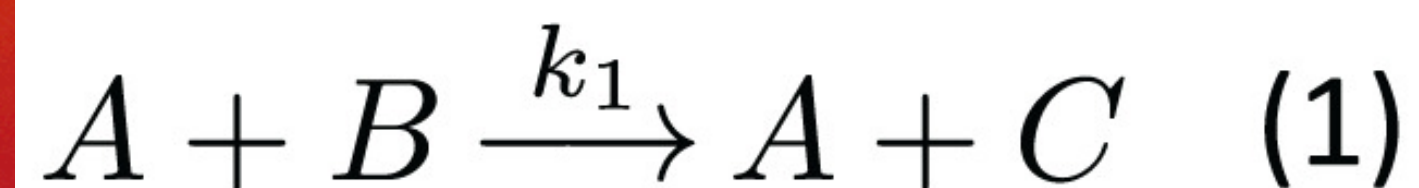
Gordon and McBride, 1994

Chemistry

Using kinetics



Chemical reaction network:



Mass-action kinetics:

$$\dot{a} = -k_2ac$$

$$\dot{b} = -k_1ab + 2k_3d$$

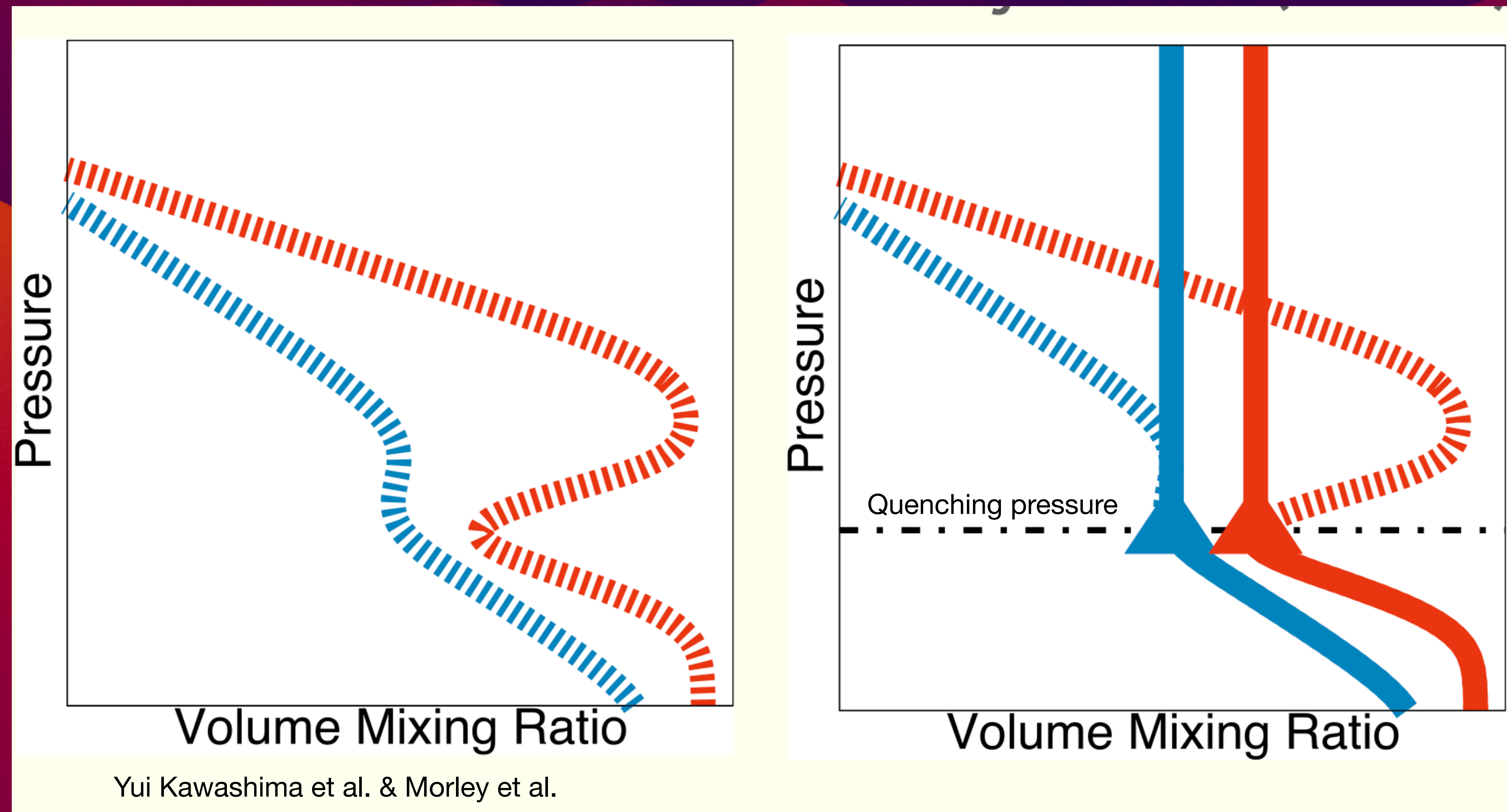
$$\dot{c} = k_1ab - k_2ac$$

$$\dot{d} = k_2ac - k_3d$$

Stoichiometry matrix:

	(1)	(2)	(3)
A	0	-1	0
B	-1	0	2
C	1	-1	0
D	0	1	-1

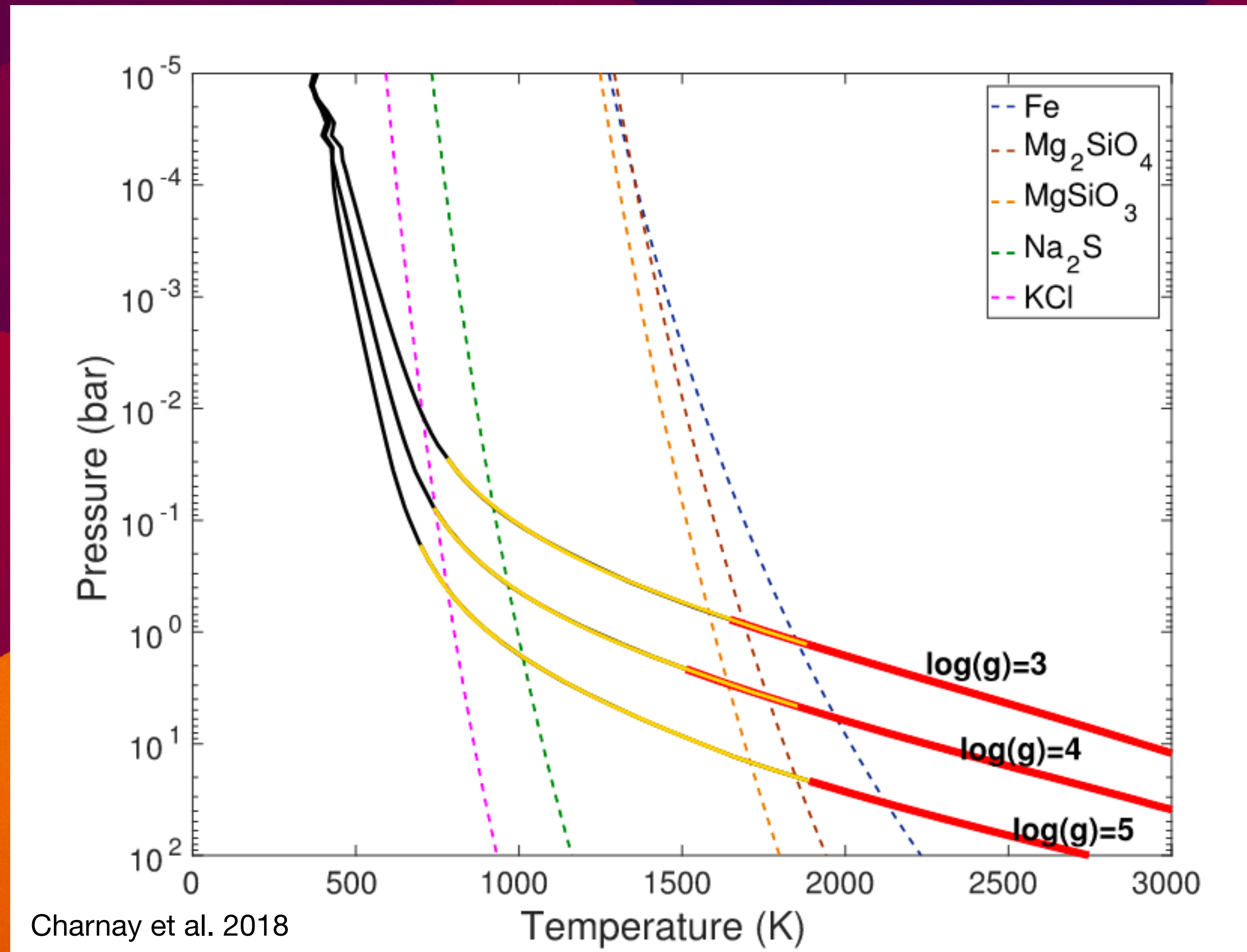
Disequilibrium Chemistry



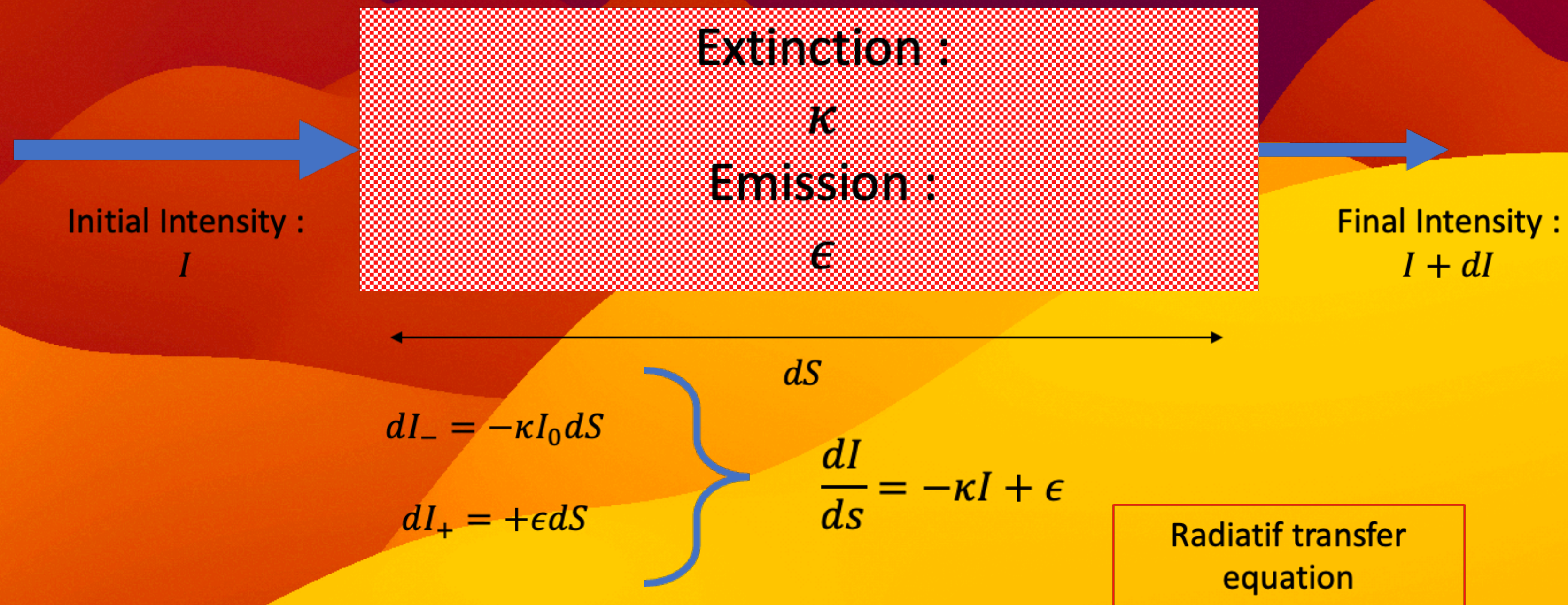
$$\tau_{chem} < \tau_{mixing}$$

$$\tau_{chem} > \tau_{mixing}$$

Cloud distribution



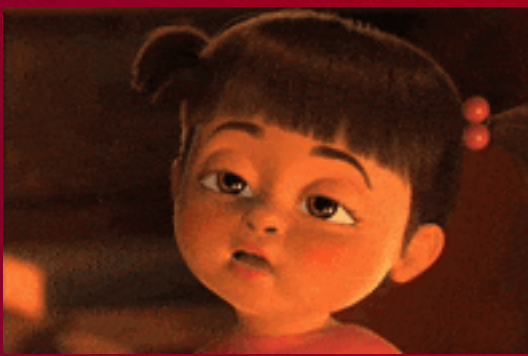
Radiative Transfer



λ dependant

$$F = \int_{\Omega} I(x, \vec{n}, \lambda, t) \vec{n} \cdot \vec{k} d\Omega = \iint I(x, \theta, \varphi, \lambda, t) \cos(\theta) \sin(\theta) d\theta d\varphi$$

Radiative Transfer



- Extinction coefficient :
(Average distance traveled by a photo before being scattered)⁻¹

$$\tau_\lambda = \int \kappa_\lambda dS$$

- Source function :

$$S_\lambda = \frac{\epsilon_\lambda}{\kappa_\lambda}$$

- Local thermodynamic equilibrium:

$$S_\lambda = B_\lambda$$

$$\frac{dI}{ds} = -\kappa I + \epsilon$$

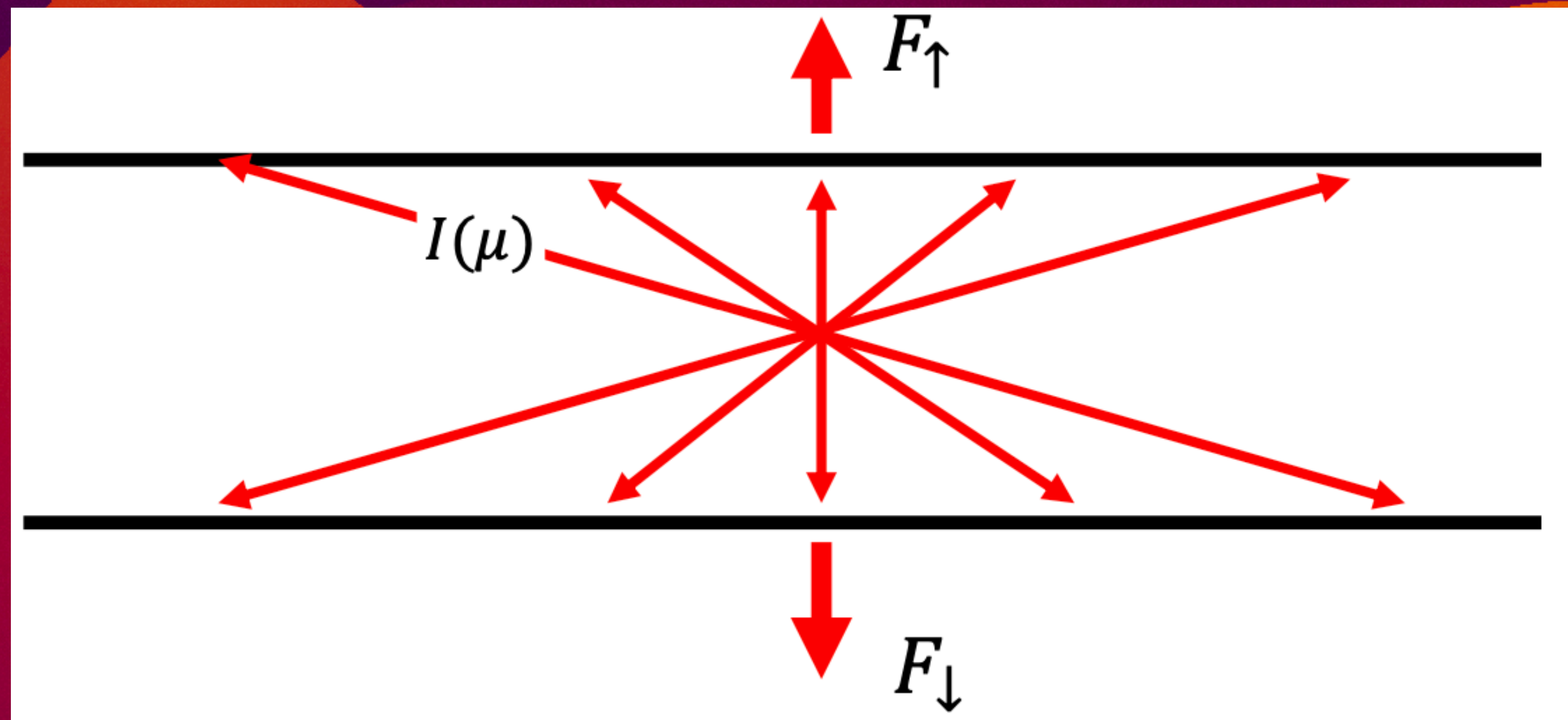
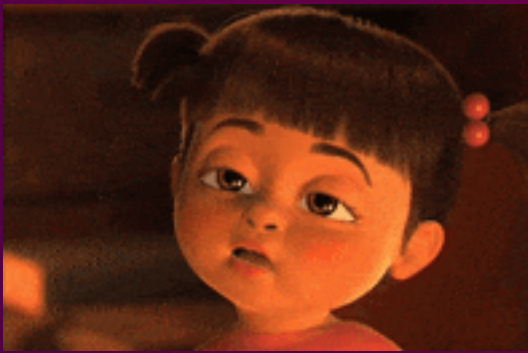


$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + B_\lambda$$

$$B(T, \lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Radiative Transfer

Two stream approximation



$$\begin{aligned}\frac{\partial F^{\uparrow}}{\partial \tau} &= \gamma_1 F^{\uparrow} - \gamma_2 F^{\downarrow} - 2\pi(1 - \omega_0)B \\ \frac{\partial F^{\downarrow}}{\partial \tau} &= \gamma_2 F^{\uparrow} - \gamma_1 F^{\downarrow} + 2\pi(1 - \omega_0)B\end{aligned}$$

Thermal profile

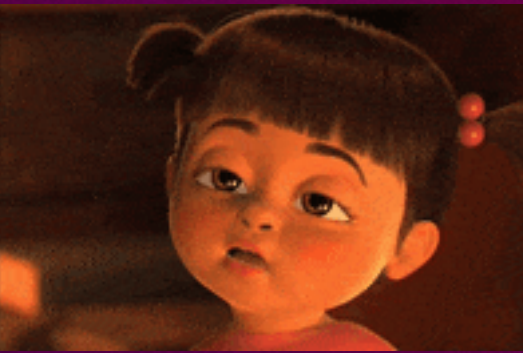
Methods for solving RT equation

$$\frac{\delta F}{F} \approx - \frac{T}{B} \frac{\partial B}{\partial T} \frac{d \ln T}{d \ln P} \frac{\delta k}{k}$$

1. Gray/Semi-Gray analytical model (Guillot et al. / Parmentier et al.)
2. Correlated-k method

Atmospheric dynamics

Euler equations (Boussinesq regime)



$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u} \otimes \vec{u} + P) = \rho \vec{g}$$

$$\frac{\partial \rho X}{\partial t} + \vec{\nabla} \cdot (\rho X \vec{u}) = \rho R(X, P, T)$$

$$\frac{\partial \rho \epsilon}{\partial t} + \vec{\nabla} \cdot (\vec{u} (\rho \epsilon + P)) = \rho c_p H(X, P, T)$$

Atmospheric dynamics

Euler equations linearised



$$\exp(\omega t + i(k_x + k_y + k_z))$$

$$\vec{\nabla} \cdot (\delta \vec{u}) = 0$$

$$\frac{\partial \rho_0 \delta \vec{u}}{\partial t} + \vec{\nabla} (\delta P) - \delta \rho \vec{g} = 0$$

$$\frac{\partial \delta X}{\partial t} + \delta \vec{u} \cdot \vec{\nabla} (X_0) - R_X \delta X - R_T \delta T = 0$$

$$\frac{\partial \delta T}{\partial t} + T_0 \delta \vec{u} \cdot \vec{\nabla} (\log \theta_0) - H_X \delta X - H_T \delta T = 0$$

$$\frac{\delta \rho}{\rho_0} + \frac{\delta T}{T_0} - \frac{\partial \log \mu_0}{\partial X} \delta X = 0$$

$$k_x \delta u + k_y \delta v + k_z \delta w = 0$$

$$\omega \rho_0 \delta u + i k_x \delta P = 0$$

$$\omega \rho_0 \delta v + i k_y \delta P = 0$$

$$\omega \rho_0 \delta w + i k_z \delta P + \delta \rho g = 0$$

$$\omega \delta X + \delta w \frac{\partial X_0}{\partial z} - R_X \delta X - R_T \delta T = 0$$

Atmospheric dynamics

Euler equations : Jacobien



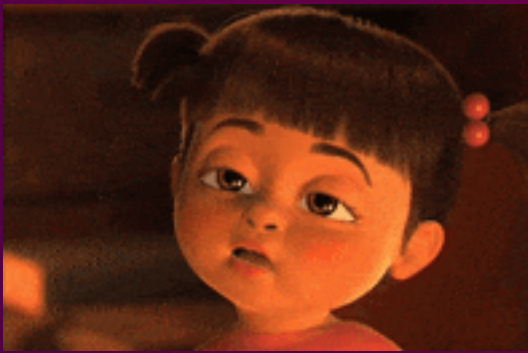
$$\begin{aligned}
 ik^2 \delta P + k_z \delta \rho g &= 0 \\
 \omega \rho_0 \delta w + ik_z \delta P + \delta \rho g &= 0 \\
 \omega \delta X + \delta w \frac{\partial X_0}{\partial z} - R_X \delta X - R_T \delta T &= 0 \\
 \omega \delta T - \delta w \frac{T_0}{h_p} (\nabla_T - \nabla_{\text{ad}}) - H_X \delta X - H_T \delta T &= 0 \\
 \frac{\delta \rho}{\rho_0} + \frac{\delta T}{T_0} - \frac{\partial \log \mu_0}{\partial X} \delta X &= 0
 \end{aligned}$$

$$\begin{pmatrix}
 k_z g & 0 & 0 & 0 & ik^2 \\
 0 & \omega - R_X & \frac{\partial X_0}{\partial z} & -R_T & 0 \\
 g & 0 & \omega \rho_0 & 0 & ik_z \\
 0 & -H_X & -\frac{T_0}{h_p} (\nabla_T - \nabla_{\text{ad}}) & \omega - H_T & 0 \\
 -1/\rho_0 & \frac{\partial \log \mu_0}{\partial X} & 0 & -1/T_0 & 0
 \end{pmatrix}$$

$$\nabla_T = \frac{\partial \log(T)}{\partial \log(P)} \quad \nabla_{\text{ad}} = \frac{\gamma - 1}{\gamma}$$

Atmospheric dynamics

Schwarzschild criterion



$$H_T = 0 \quad R_X = 0$$

Stability analysis using the determinant of the Jacobien

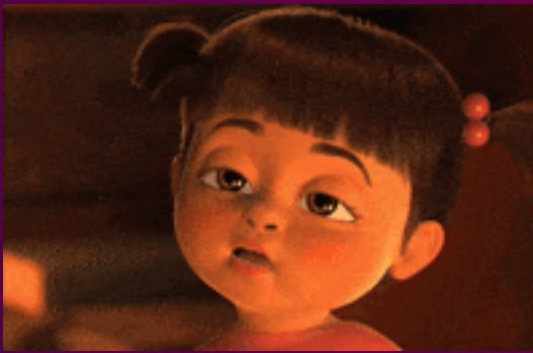
$$k_z g i k_z \frac{g}{h_p} (\nabla_T - \nabla_{ad}) - i k^2 \frac{g}{h_p} (\nabla_T - \nabla_{ad}) + i k^2 \omega^2 = 0$$
$$\omega^2 - (\nabla_T - \nabla_{ad}) \frac{k_x^2 + k_y^2}{k^2} \frac{g}{h_p} = 0$$

$$\nabla_T - \nabla_{ad} > 0$$

If validated, then convection is the most efficient heat transfer process

Atmospheric dynamics

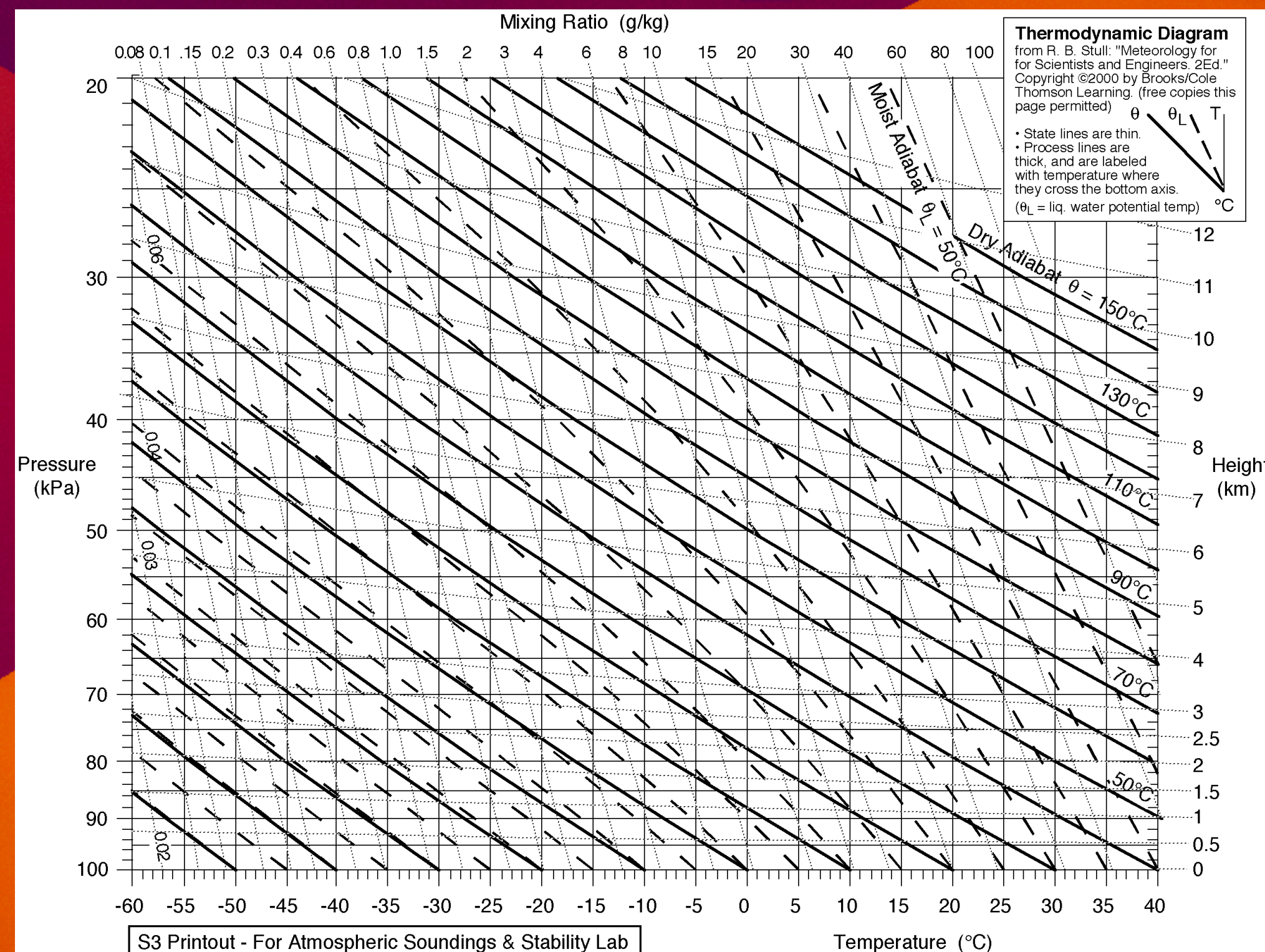
Validation of Schwarzschild criterion



$$P^{1-\gamma} T^{\gamma} = cste$$



$$\frac{d \log(T)}{d \log(P)} \approx \frac{\gamma - 1}{\gamma}$$

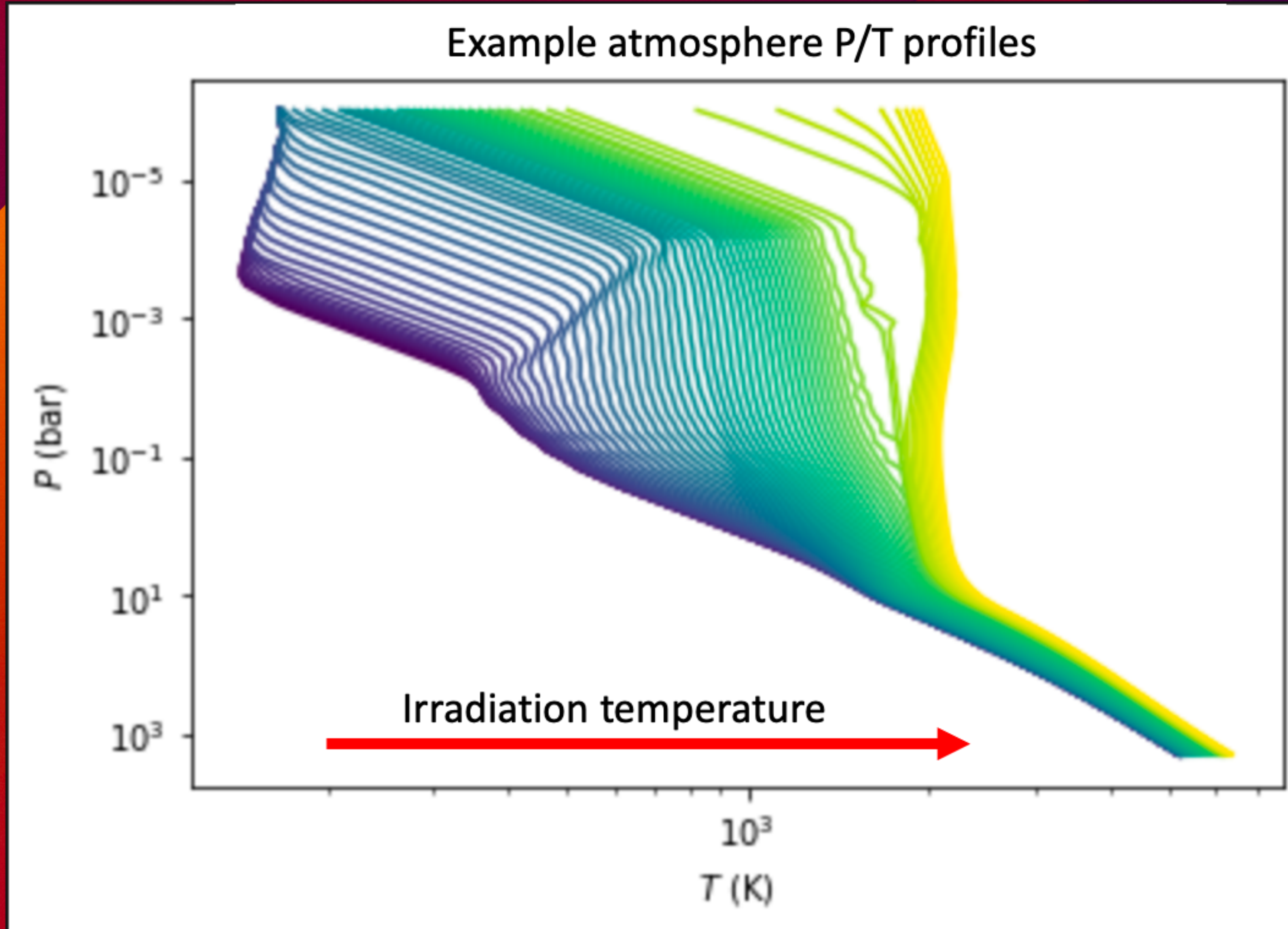


$$\frac{dT}{dz} = - \frac{\gamma - 1}{\gamma} \frac{\mu g}{R}$$

With Hydrostatic balance

Modeling atmospheres

Pressure-Temperature profiles

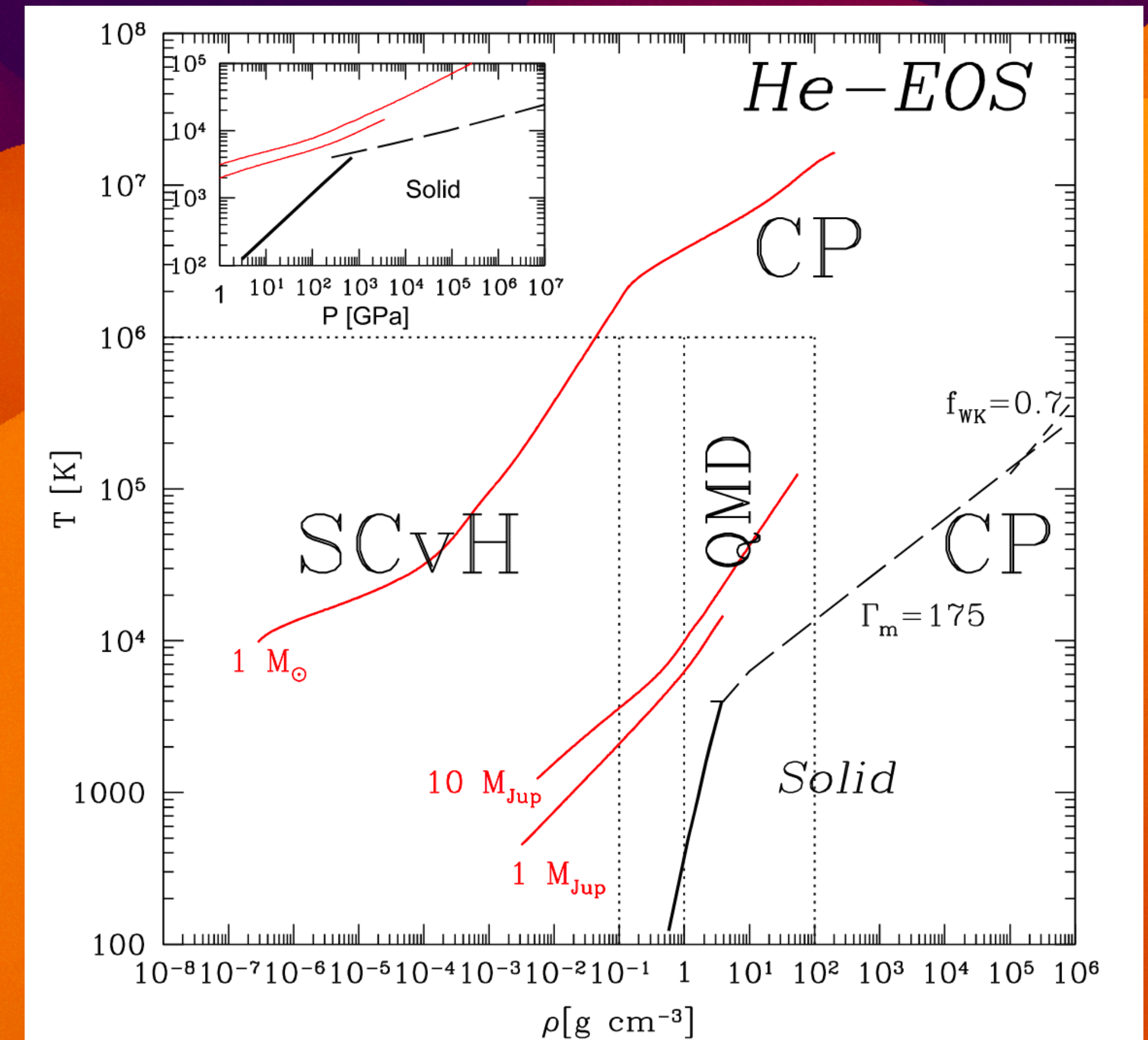
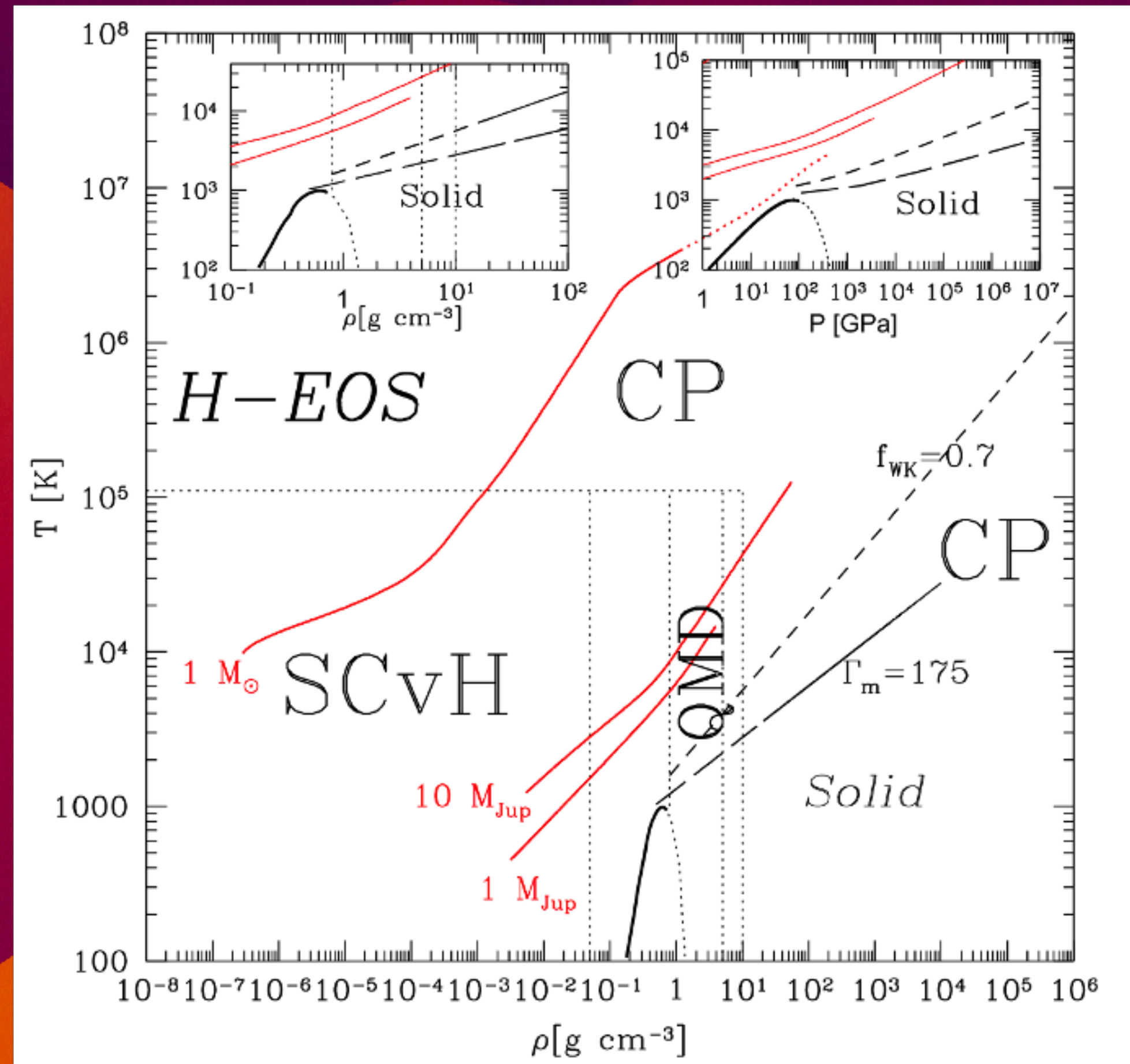


Profiles depend on :

- Chemical composition (C/O ratio, Metallicity)
- Irradiation flux
- Internal flux

Interiors

Equations of state



Mixing equations of state



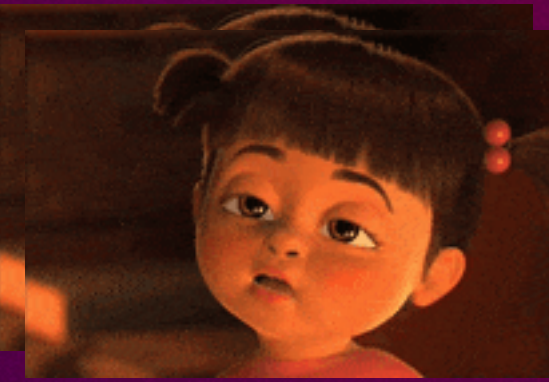
$$W(T, P) = \sum_i X_i W_i(T, P) \quad X_i = \frac{M_i}{\sum_i M_i}$$

$$S = \sum_i X_i S_i(T, P) + S_{mix}(T, P)$$

$$S = X_H S_H(T, P) + X_{He} S_{He}(T, P) + X_{H_2O} S_{H_2O}(T, P) + S_{mix}(T, P)$$

$$S_{mix}(T, P) = -k_b \frac{x_h \ln(x_h) + x_{He} \ln(x_{He})}{m_h \sum_i x_i A_i}$$

Mixing equations of state



$$\frac{1}{\rho_p} = \sum_i \frac{\alpha_i}{\rho_i} \quad \alpha_i = \frac{m_i}{m_{tot}}$$

$$\frac{1}{\rho} = \frac{\alpha_H}{\rho_H} + \frac{\alpha_{He}}{\rho_{He}} + o(\delta V)$$

Computing internal profile

Adiabatic profile

$$dS = 0$$

$$\frac{dP}{dz} = -\rho g$$

Computing internal profile

1. Define « surface » pressure and temperature

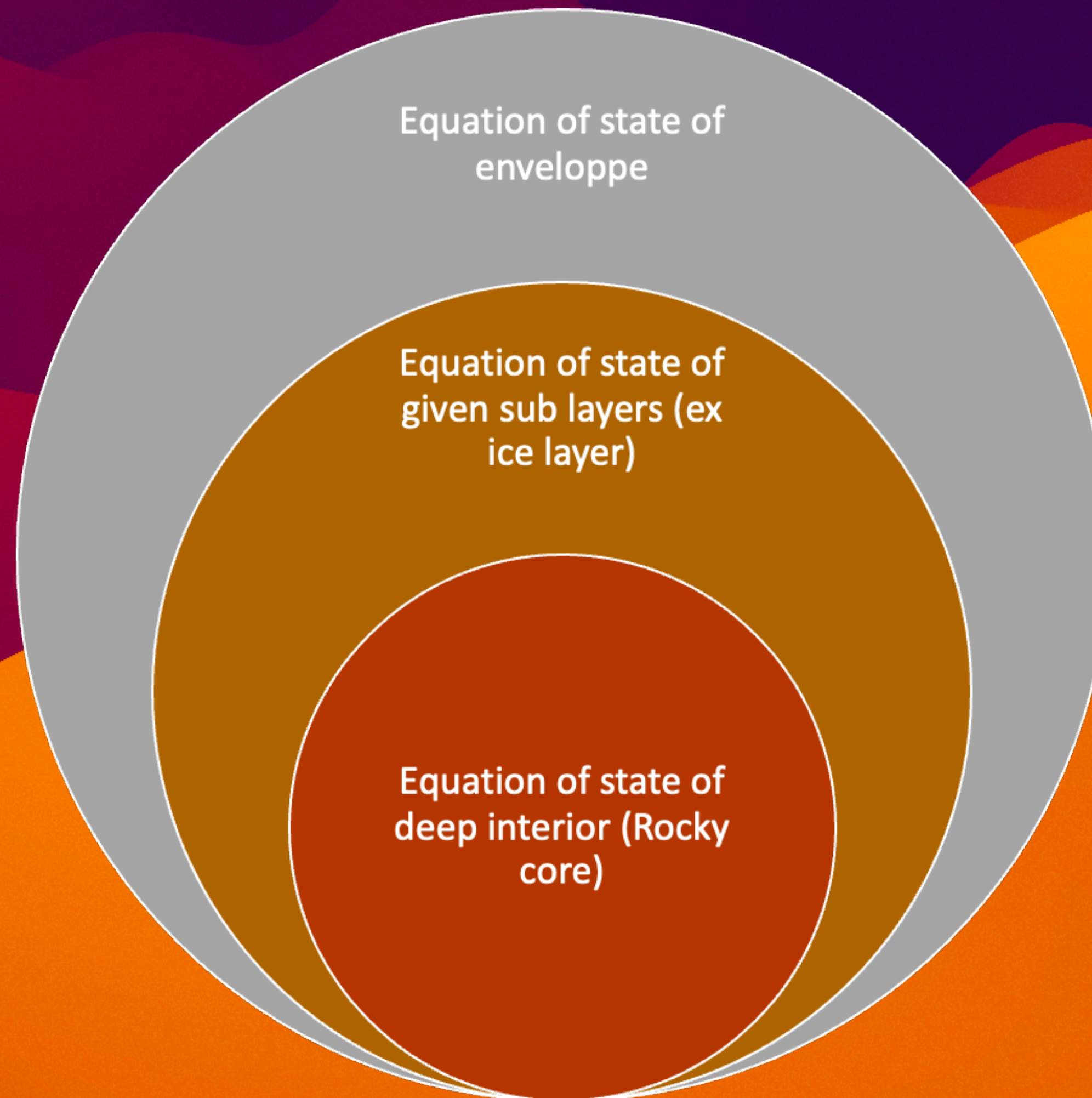
$$(P_s, T_s) = (P_0, T_0)$$

2. Evaluate entropy at « surface »

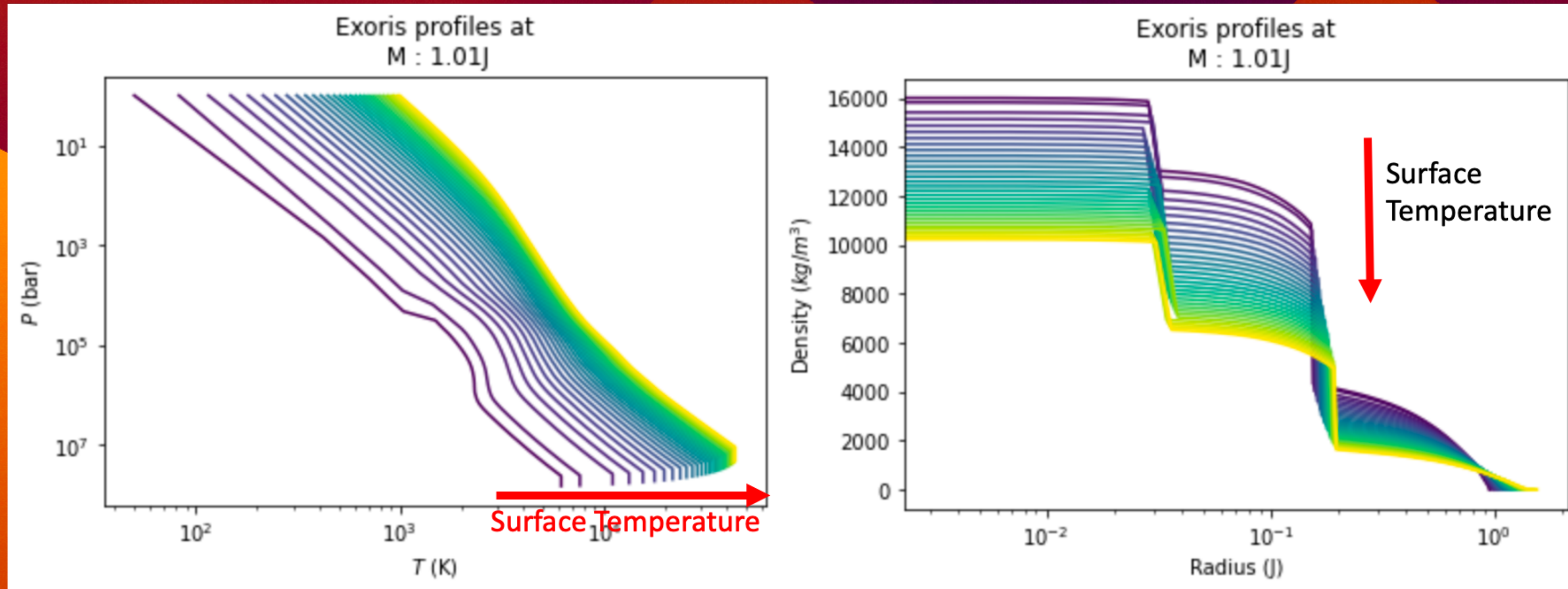
$$S = S(P_0, T_0, x_i) \ i \in [0, N_{molecules}]$$

3. Create adiabatic profile on pressure grid
4. Evaluate radial profile
5. Repeat 3 & 4 until convergence
6. Output of Planetary radius as a function of structure

Interior structure



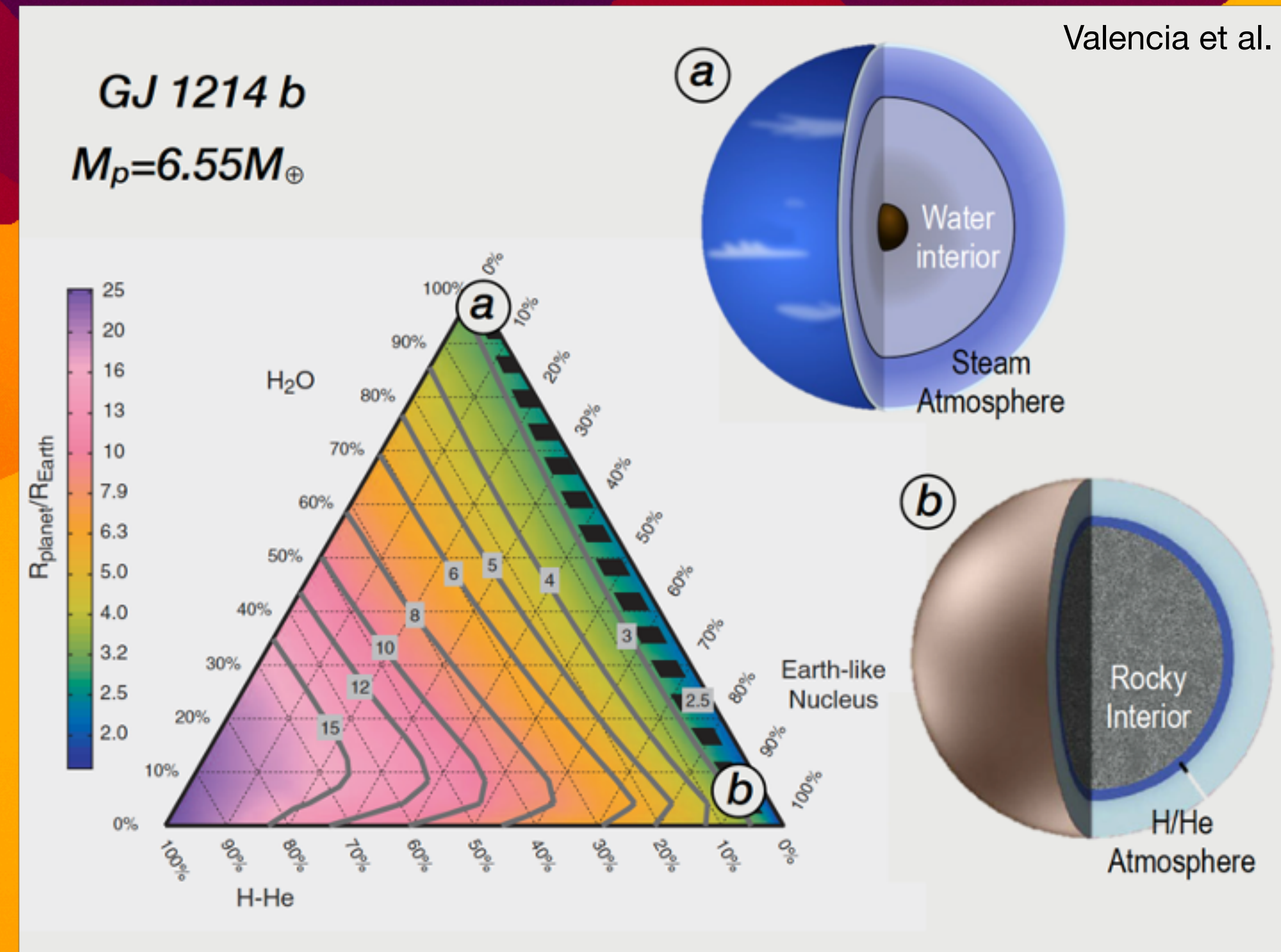
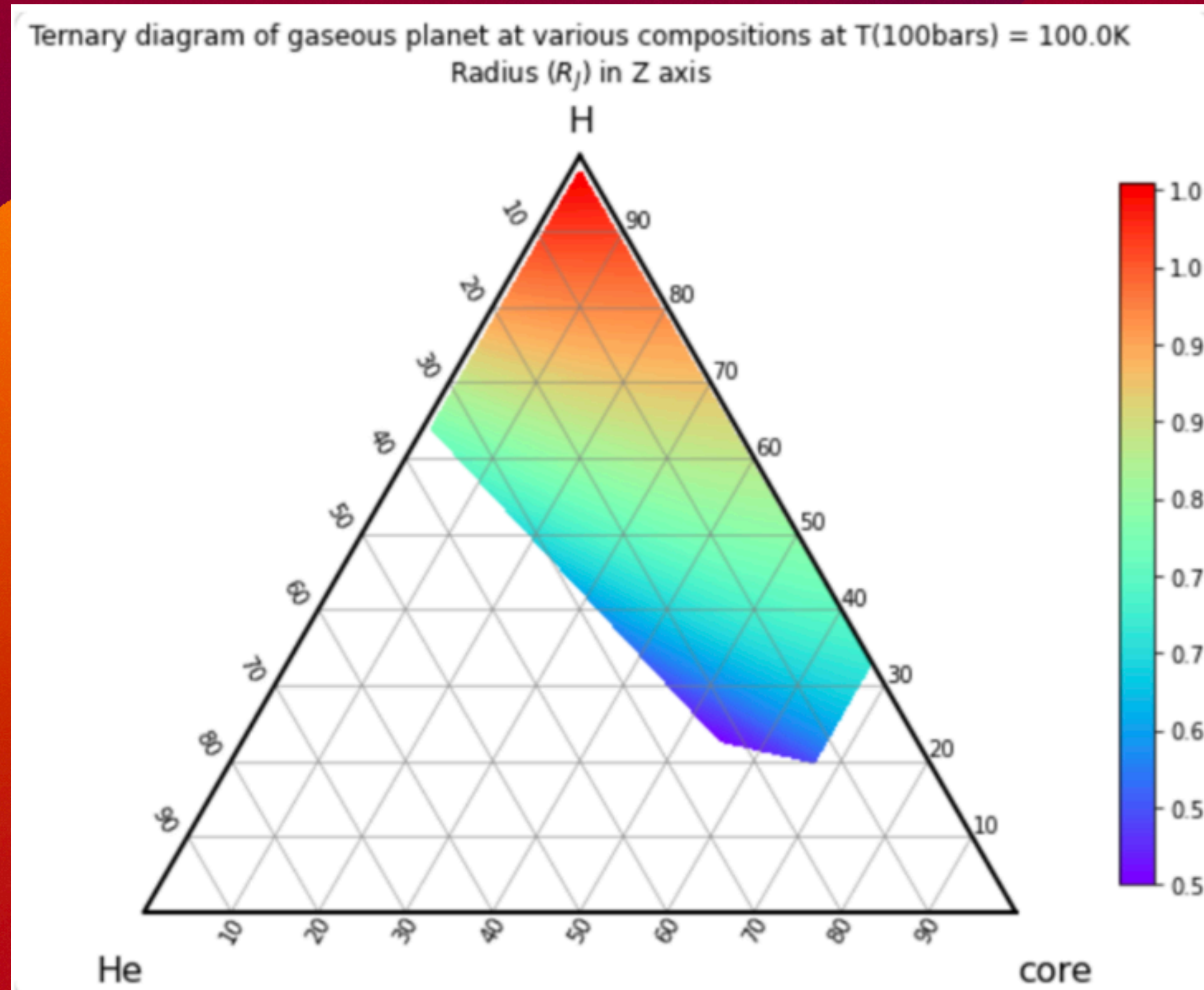
Internal profiles



Profiles depend on :

- Structure (Equations of state used)
- Core size
- H/He ratio
- Surface temperature

Interior degeneracy



The Pie Analogy



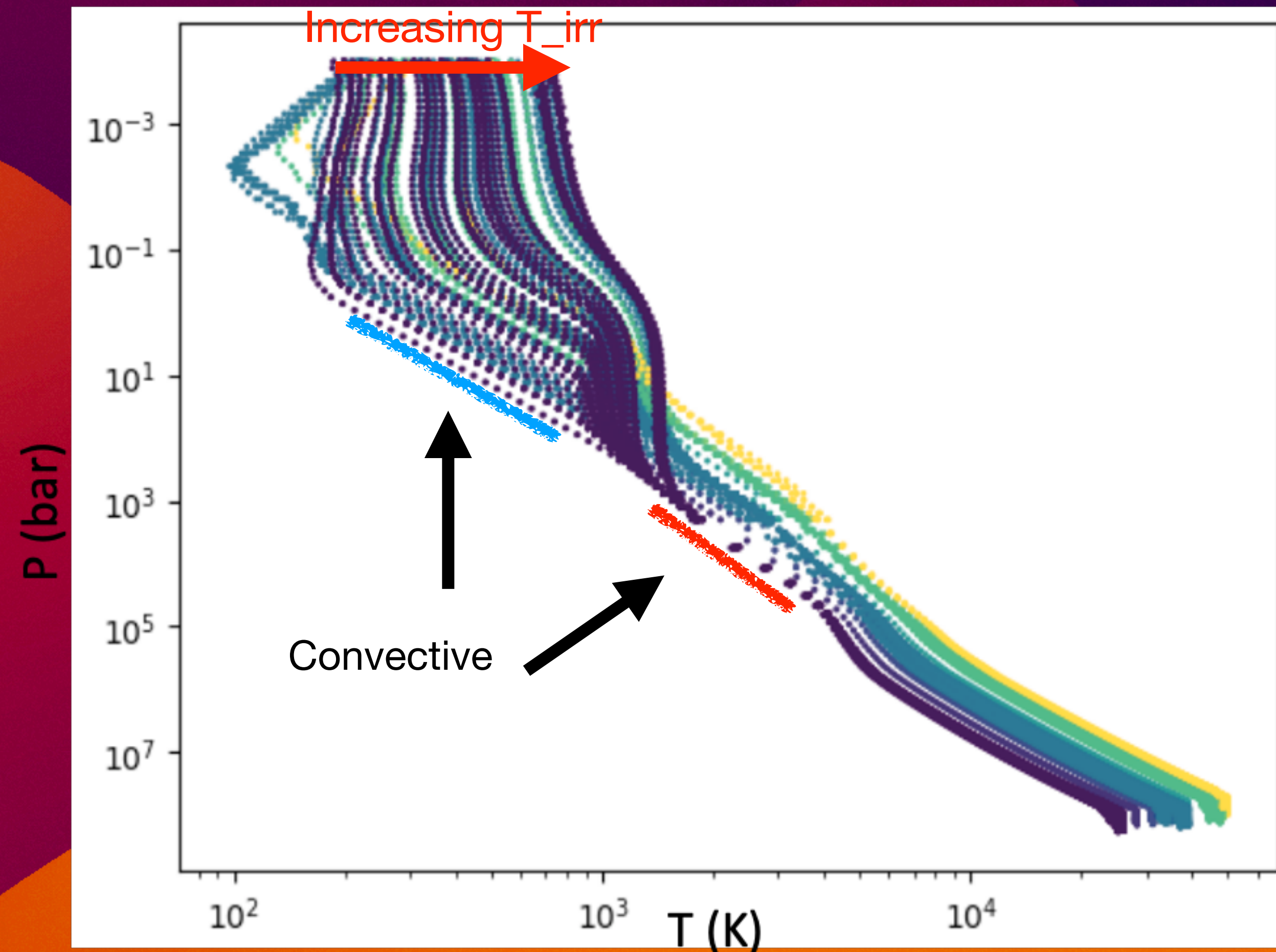
Apple pie ?
Blueberry pie ?
Pecan pie ?
Steak and kidney pie?

The Pie Analogy



We need cut
into the pie

Linking models

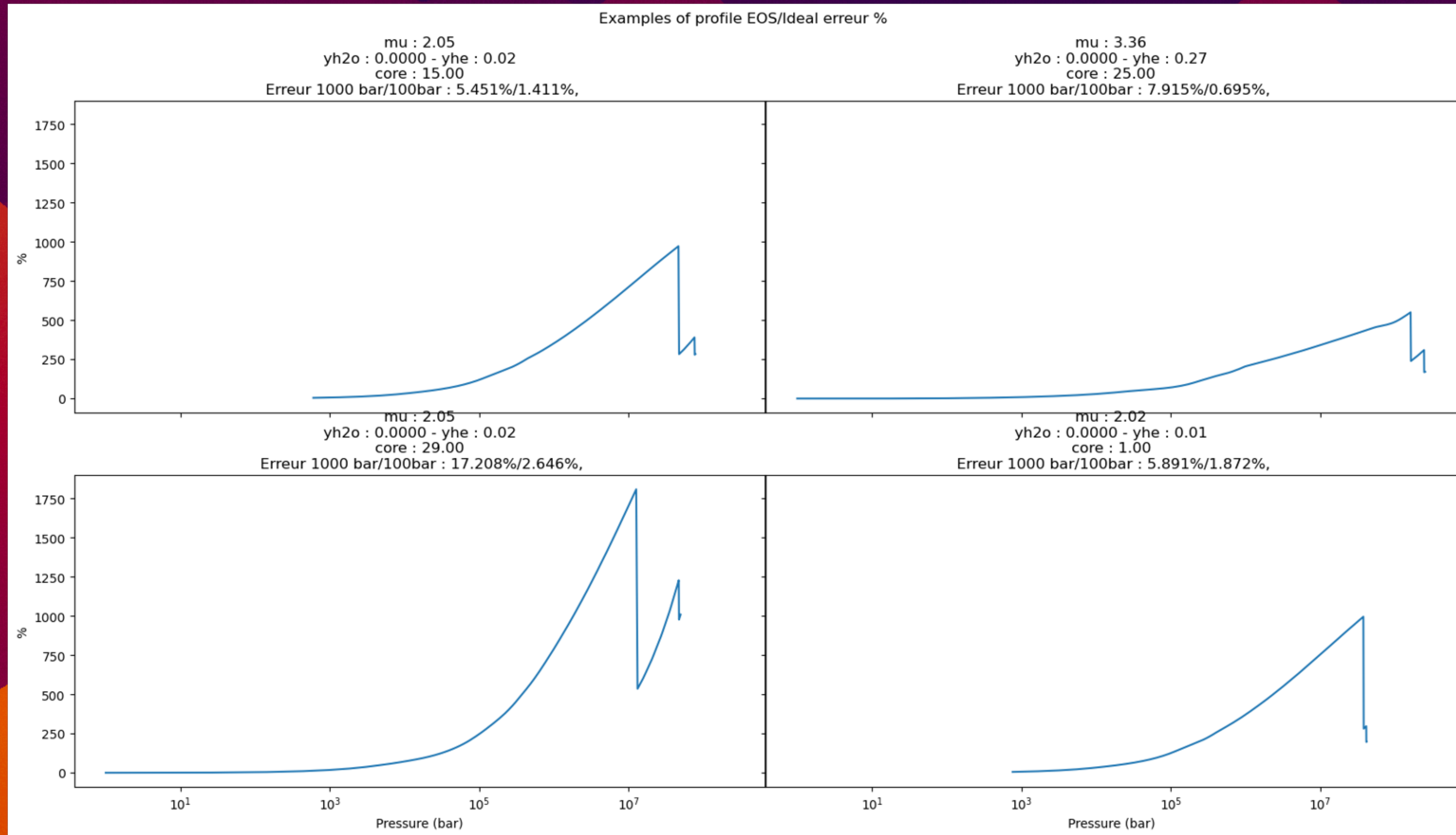


Linking :

- On Pressure and temperature
- On gravity
- On atmospheric composition (Molar mass & Density)
- On atmospheric dynamics (Convective)

Linkage errors

Ideal gas law/EOS Exoris



Linkage errors

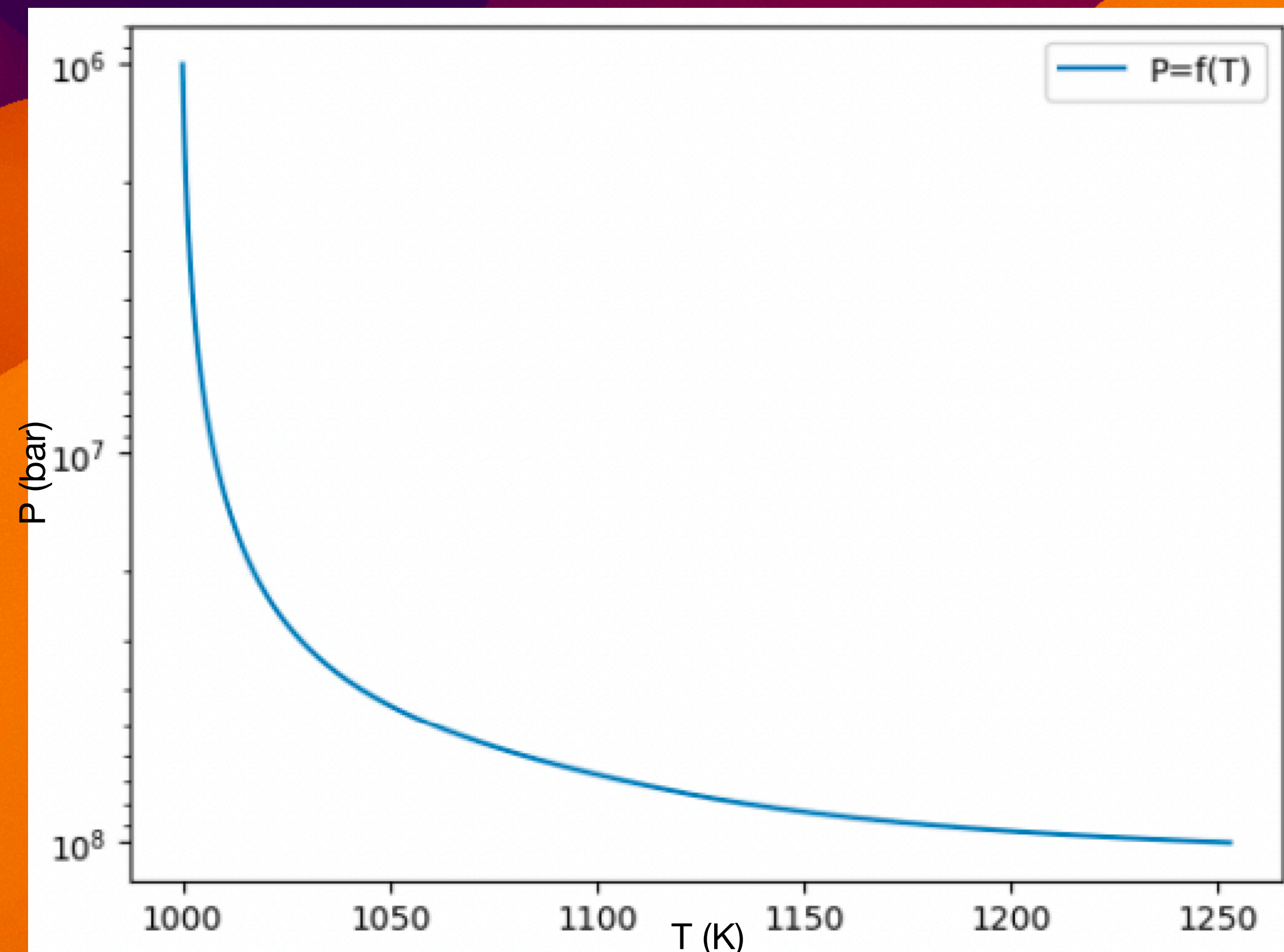
Solution - Guillot et al. 2010 - Gray analytical model

$$\frac{dT}{dz} = \frac{-3}{16} \frac{T_{int}^4}{T^3} \kappa(P, T) \text{ (Ficks law)}$$

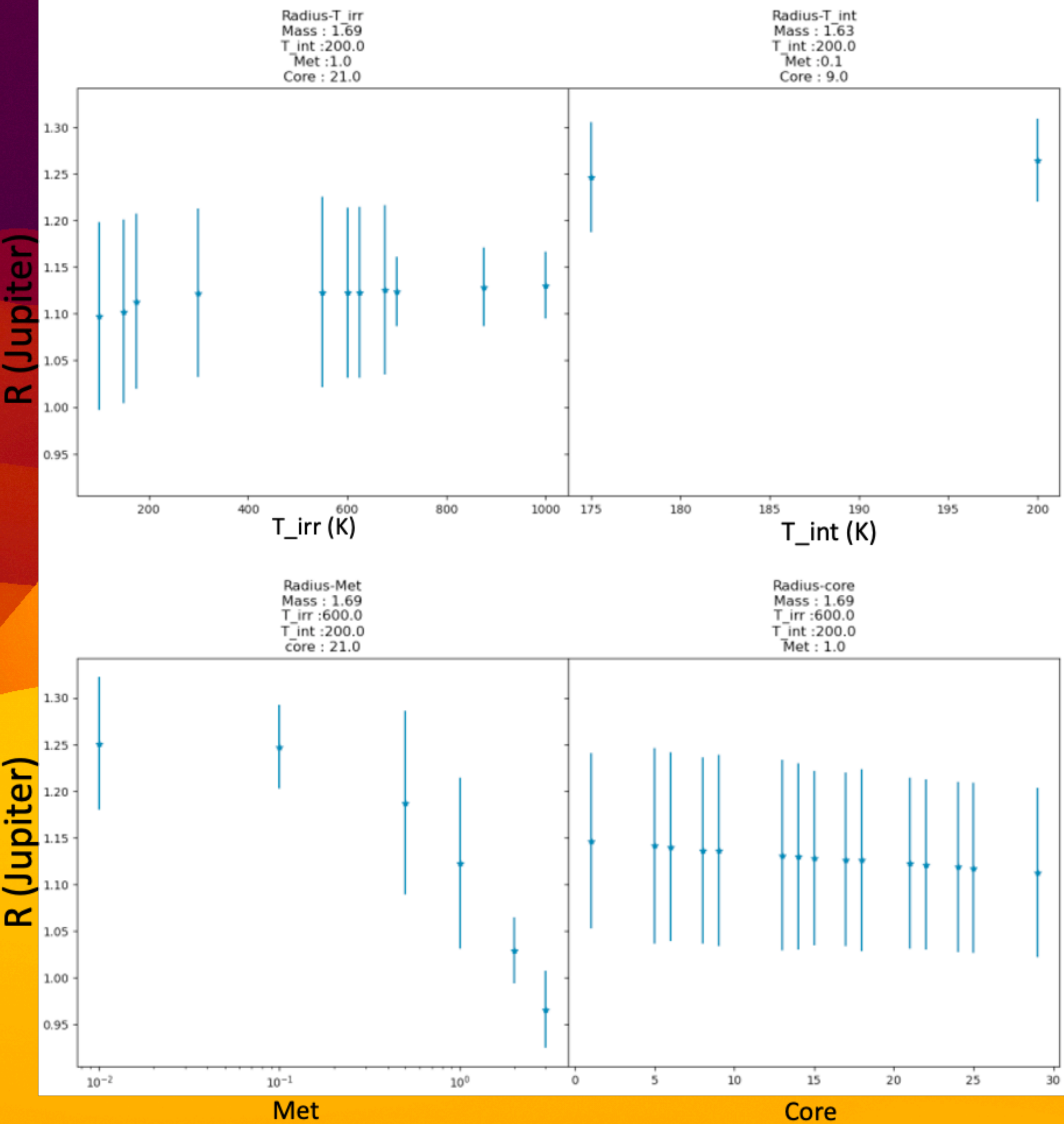
$$\frac{dP}{dz} = -\rho g \text{ (Hydrostatic balance)}$$

$$\rho = f_{exoris}(P, T) \text{ (State equation)}$$

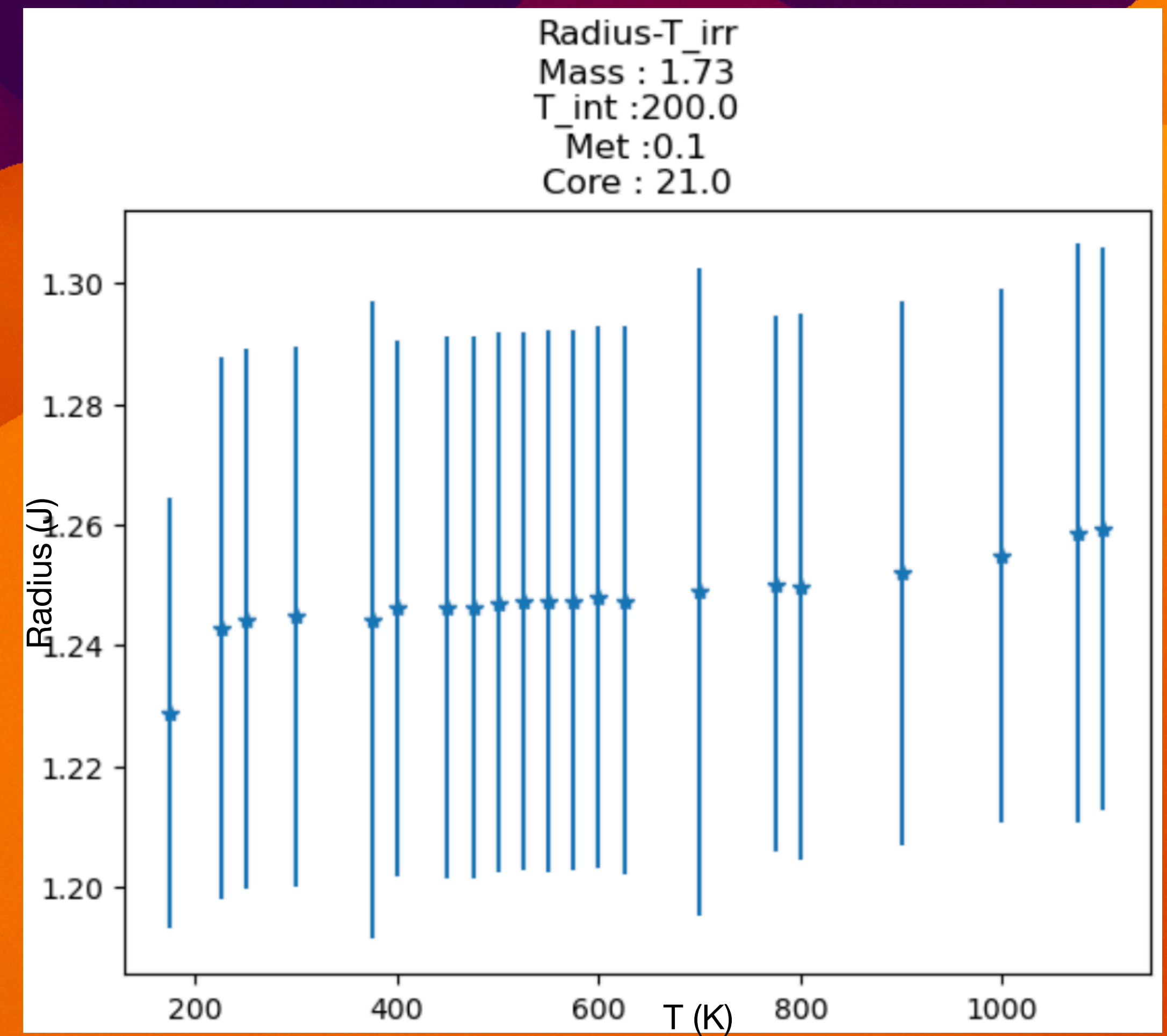
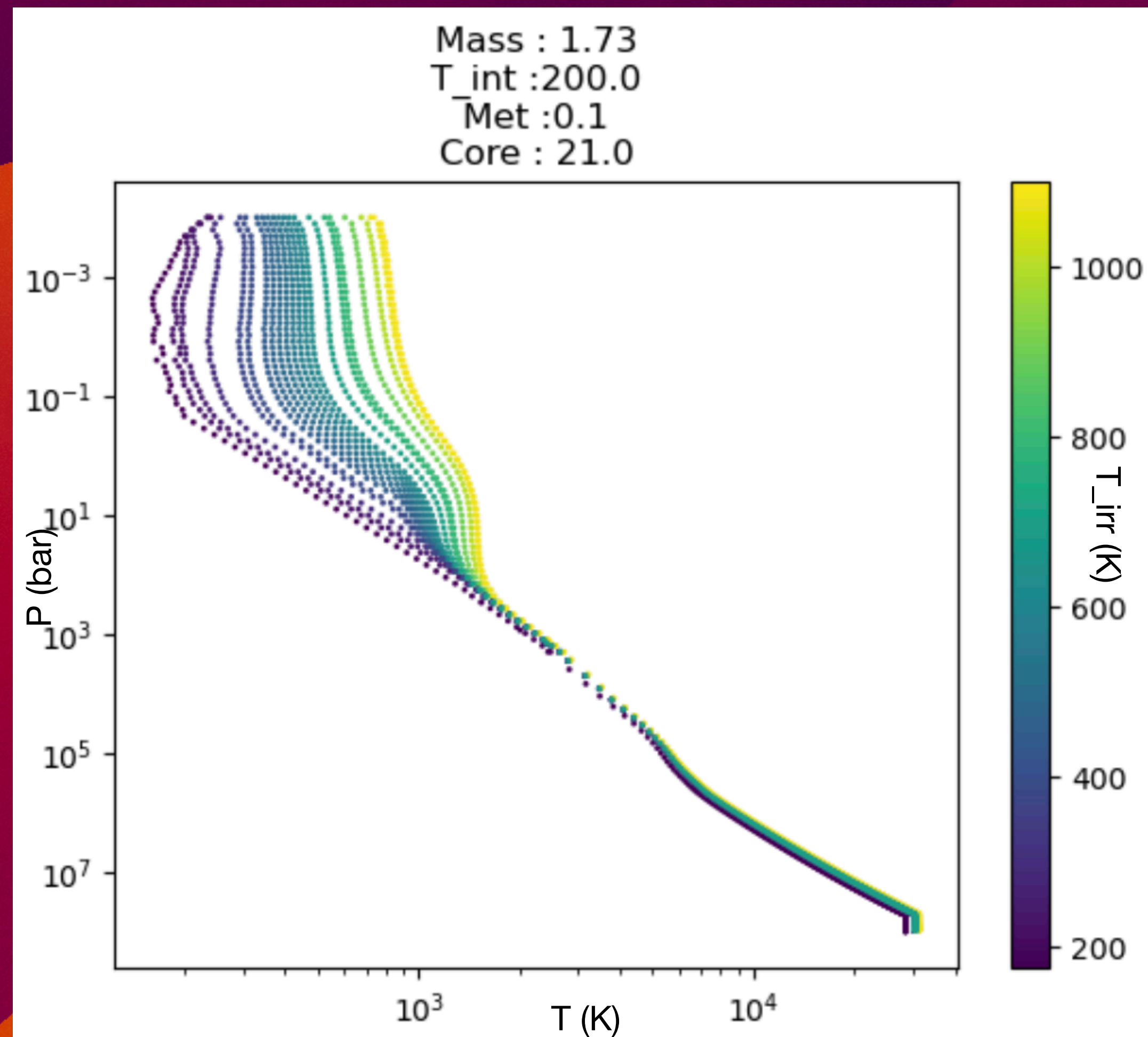
Create boundary zone between
atmosphere and interior



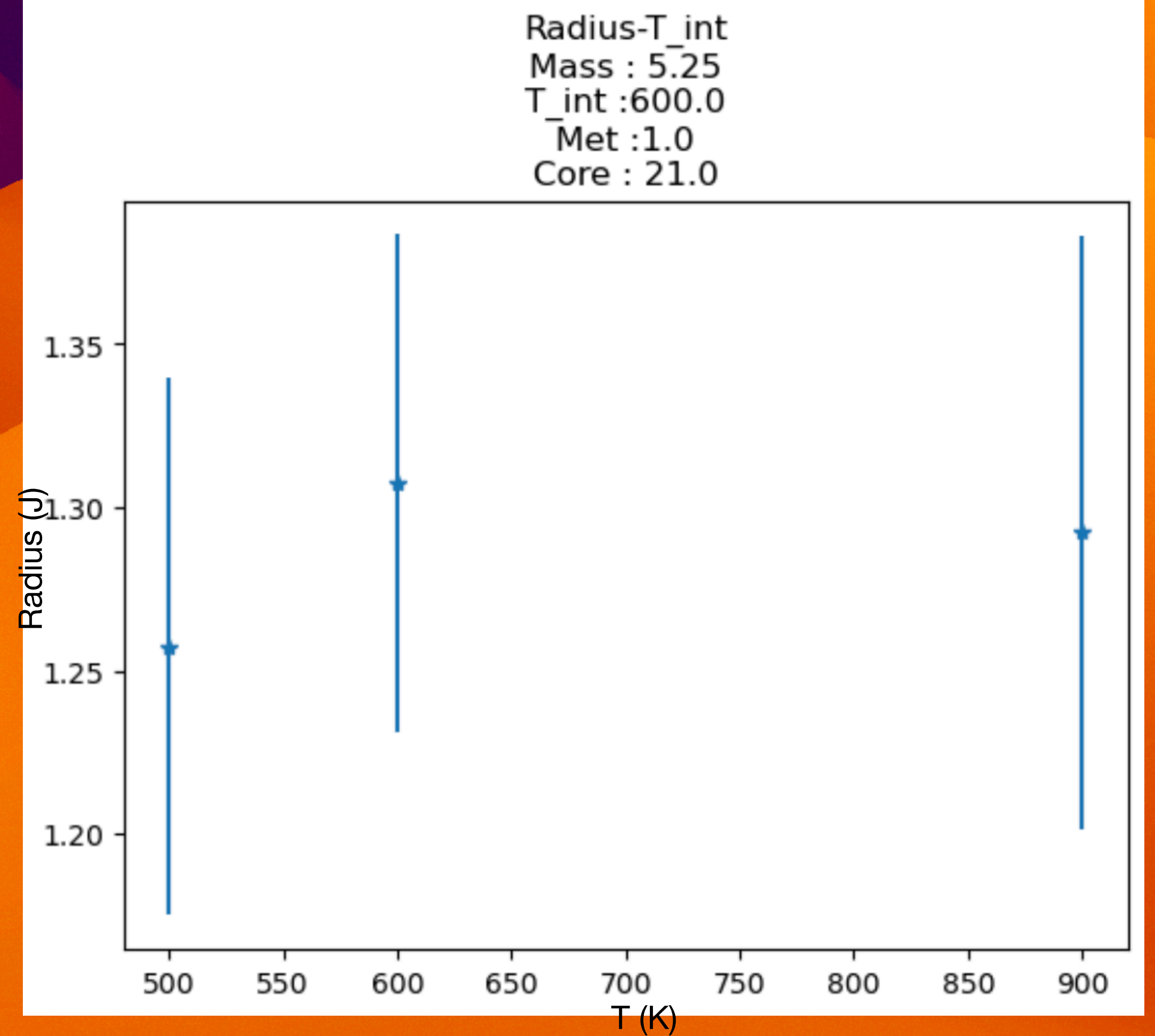
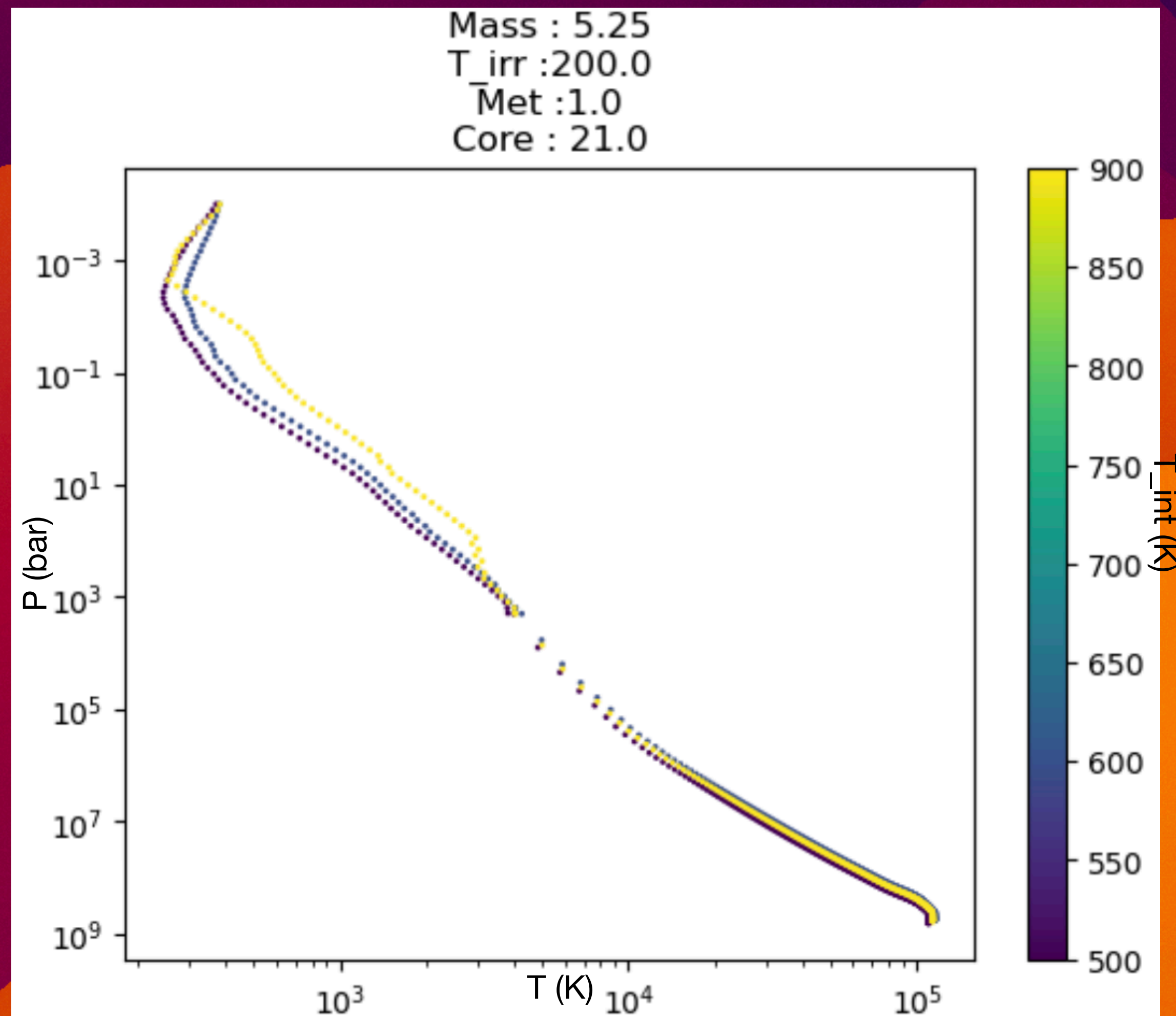
Linking models



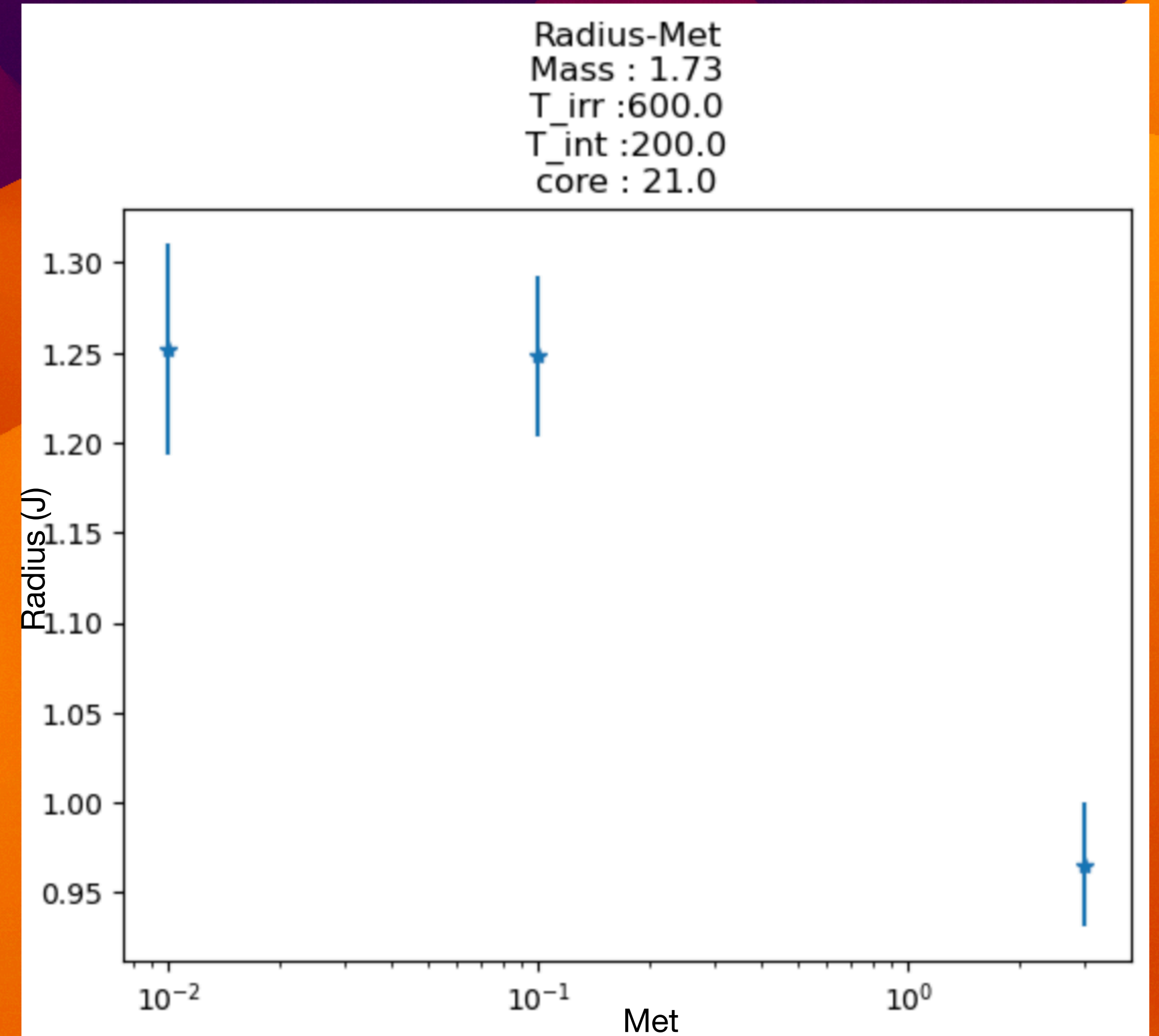
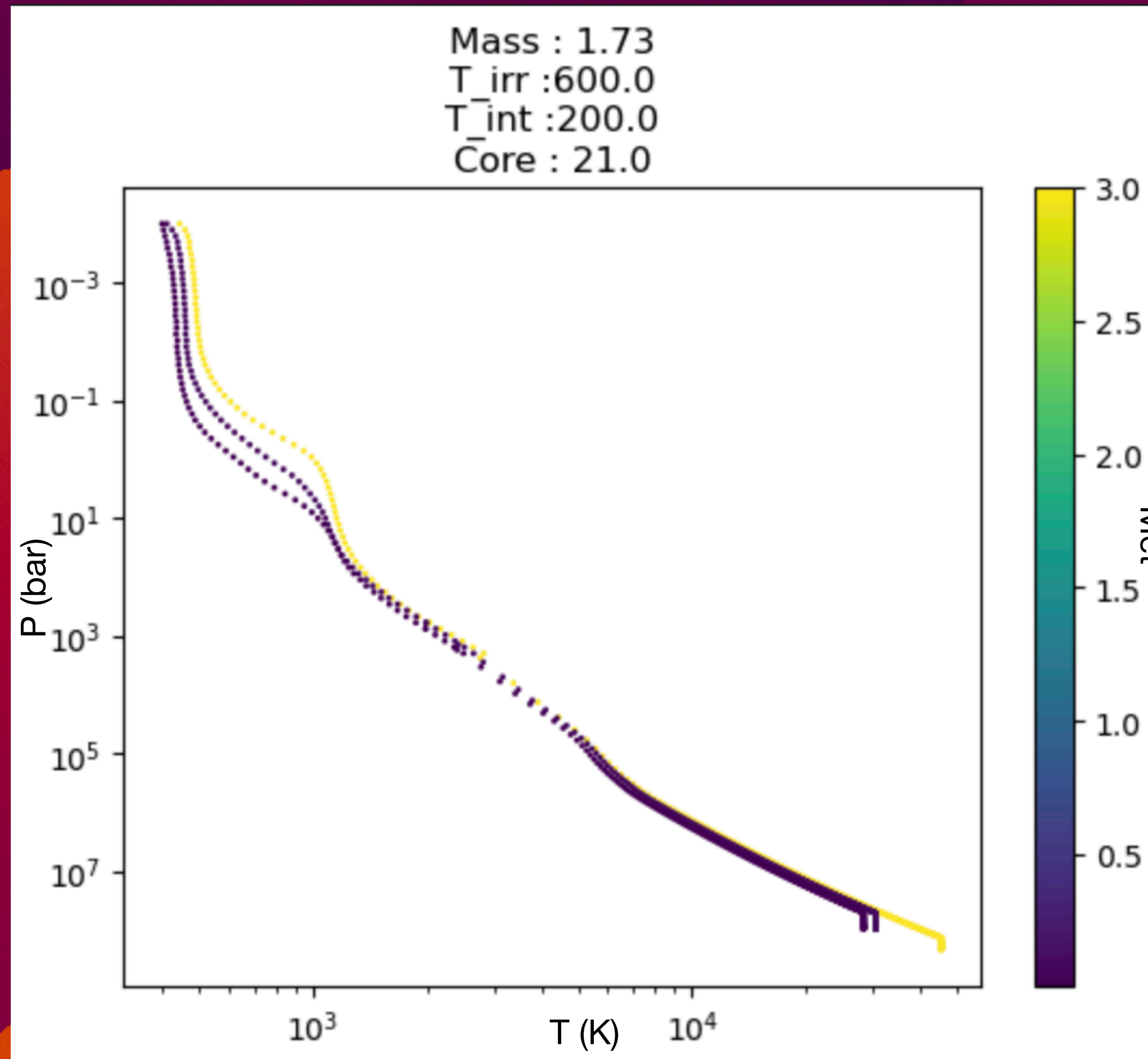
Variations with T_{irr}



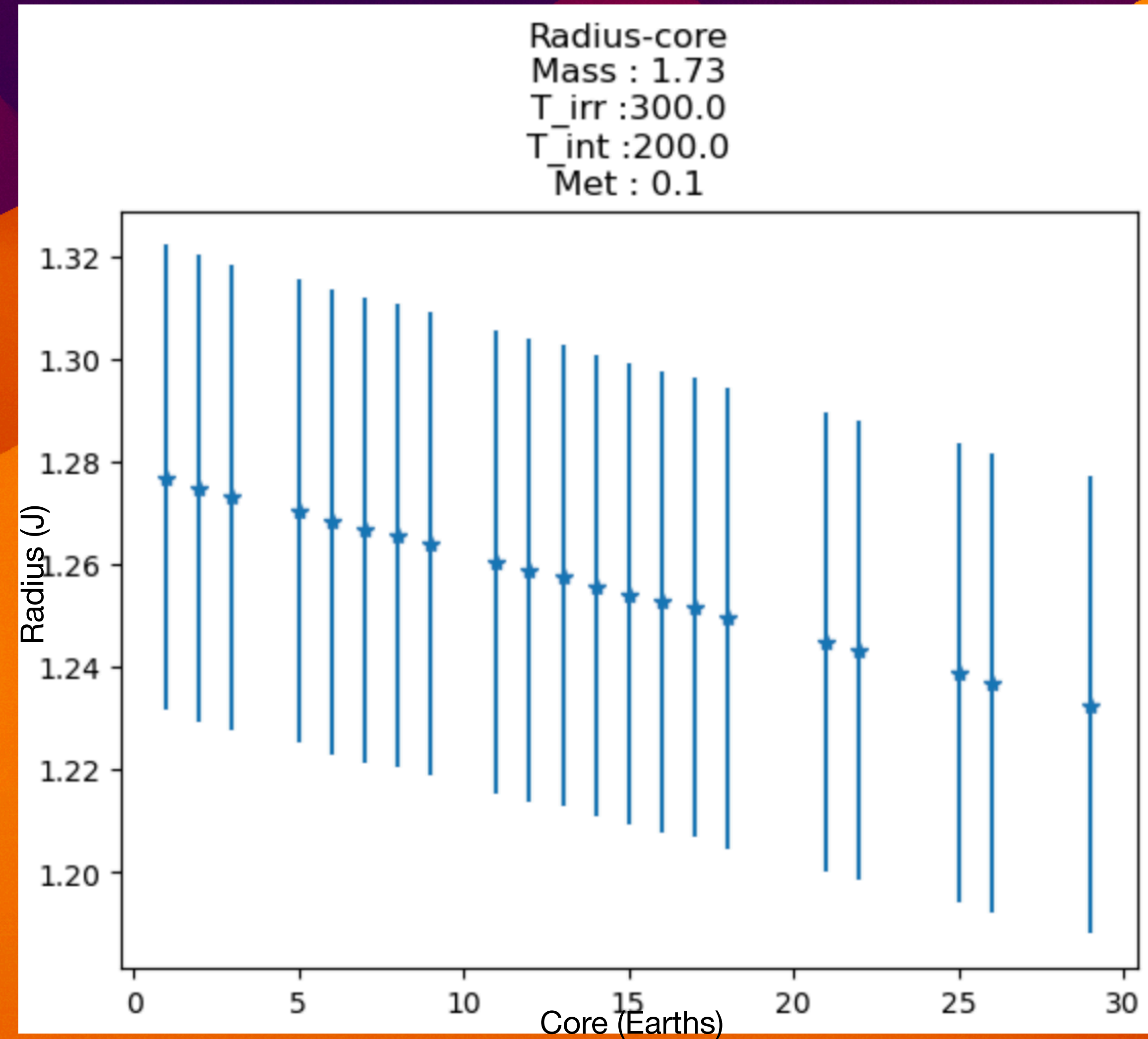
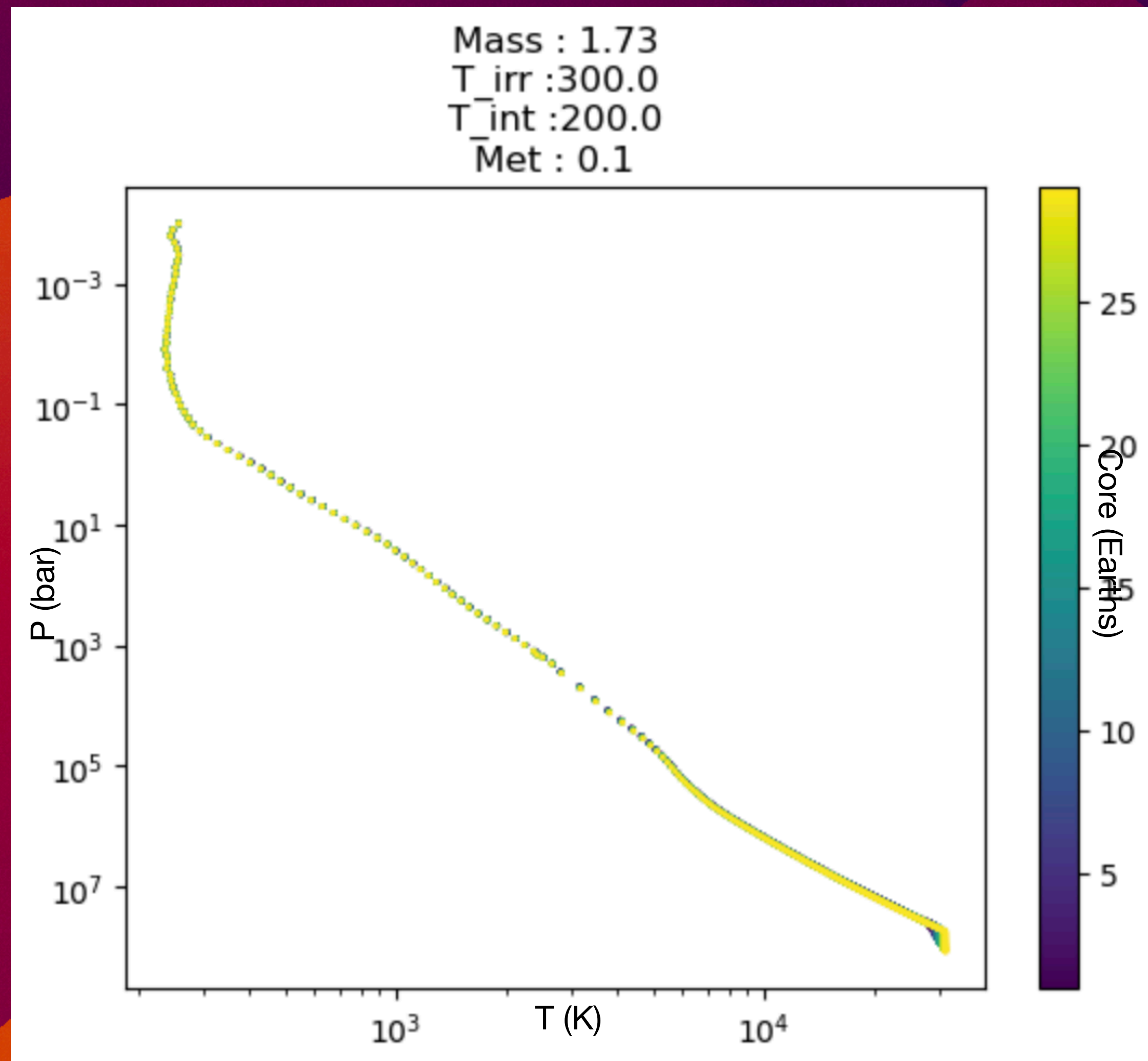
Variations with T_{int}



Variations with Met



Variations with core size



Deriving planetary age

Leconte thesis



$$\begin{aligned} l(m) &= - \int_0^m T \frac{\partial \tilde{S}}{\partial t} dm \\ &= - \int_0^m T dm \frac{\partial \tilde{S}}{\partial t} \\ &\equiv -\bar{T}(m) \frac{\partial \tilde{S}}{\partial t}, \end{aligned}$$

$$\frac{\partial}{\partial t} [\tilde{S}(T_{\text{int}})] = -4\pi \sigma_{\text{SB}} \frac{R_1^2(T_{\text{int}}) T_{\text{int}}^4}{\bar{T}_1(T_{\text{int}})}.$$

Replacing T_{int}^4 by $(T_{\text{int}}^4 + T_{\text{irr}}^4)$ will give age for irradiated planets

Atmospheric dynamics

Thermo-compositional convection Tremplin et al. 2019



$$\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0$$

$$a_2 = -R_X - H_T$$

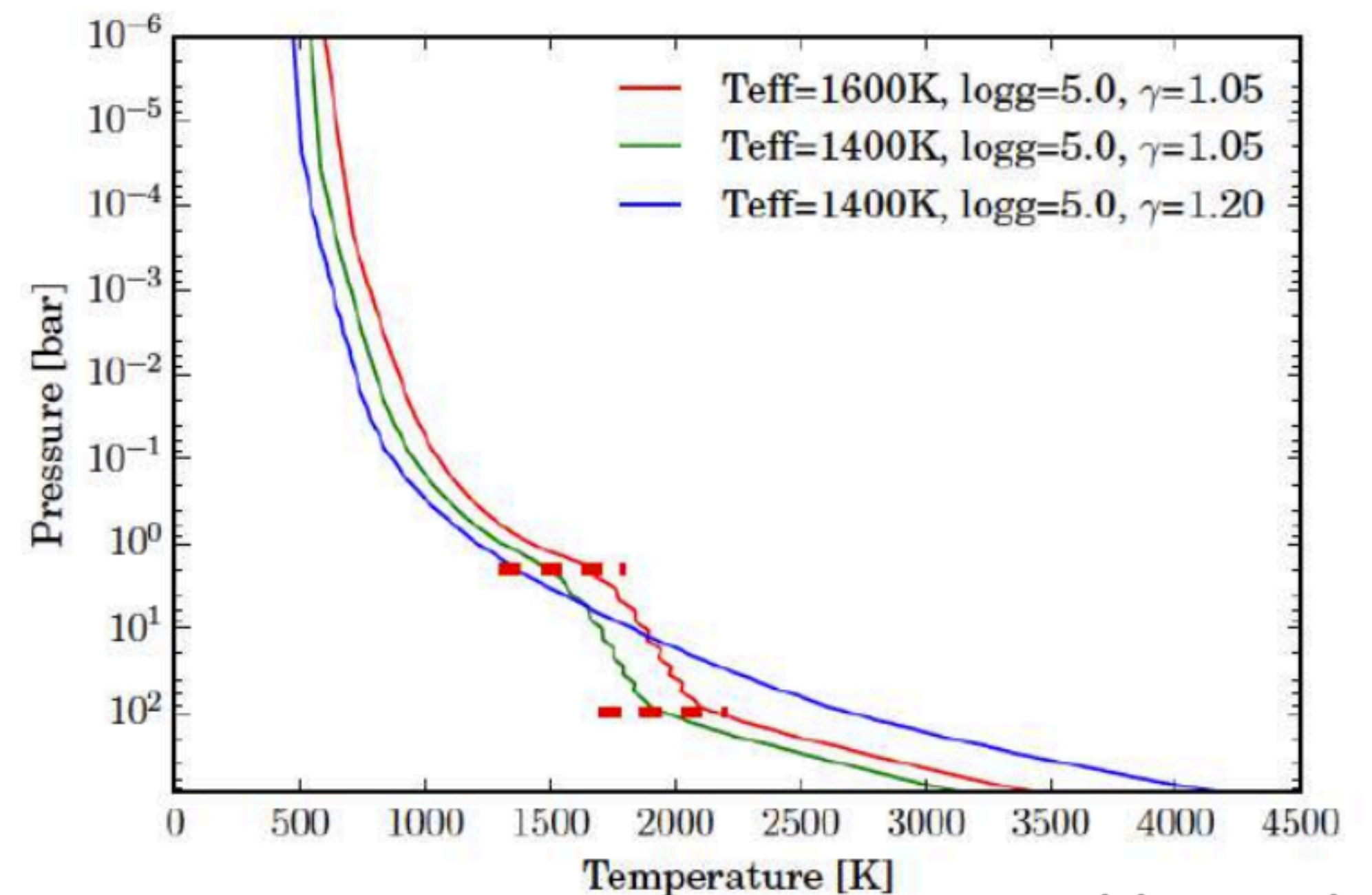
$$a_1 = H_T R_X - H_X R_T - \frac{k_x^2 + k_y^2}{k^2} \frac{g}{h_p} (\nabla_T - \nabla_{\text{ad}} - \nabla_\mu)$$

$$a_0 = \frac{k_x^2 + k_y^2}{k^2} \frac{g}{h_p} ((\nabla_T - \nabla_{\text{ad}}) R_X - \nabla_\mu H_T) \\ + \frac{k_x^2 + k_y^2}{k^2} T_0 \frac{g}{h_p} (\nabla_T - \nabla_{\text{ad}}) \frac{\partial \log \mu_0}{\partial X} R_T \\ + \frac{k_x^2 + k_y^2}{k^2} g \frac{1}{T_0} H_X \frac{\partial X_0}{\partial z}$$

$$-R_X - H_T < 0$$

$$\nabla_T - \nabla_{\text{ad}} - \nabla_\mu > (H_T R_X - H_X R_T) \frac{k^2}{k_x^2 + k_y^2} \frac{h_p}{g}$$

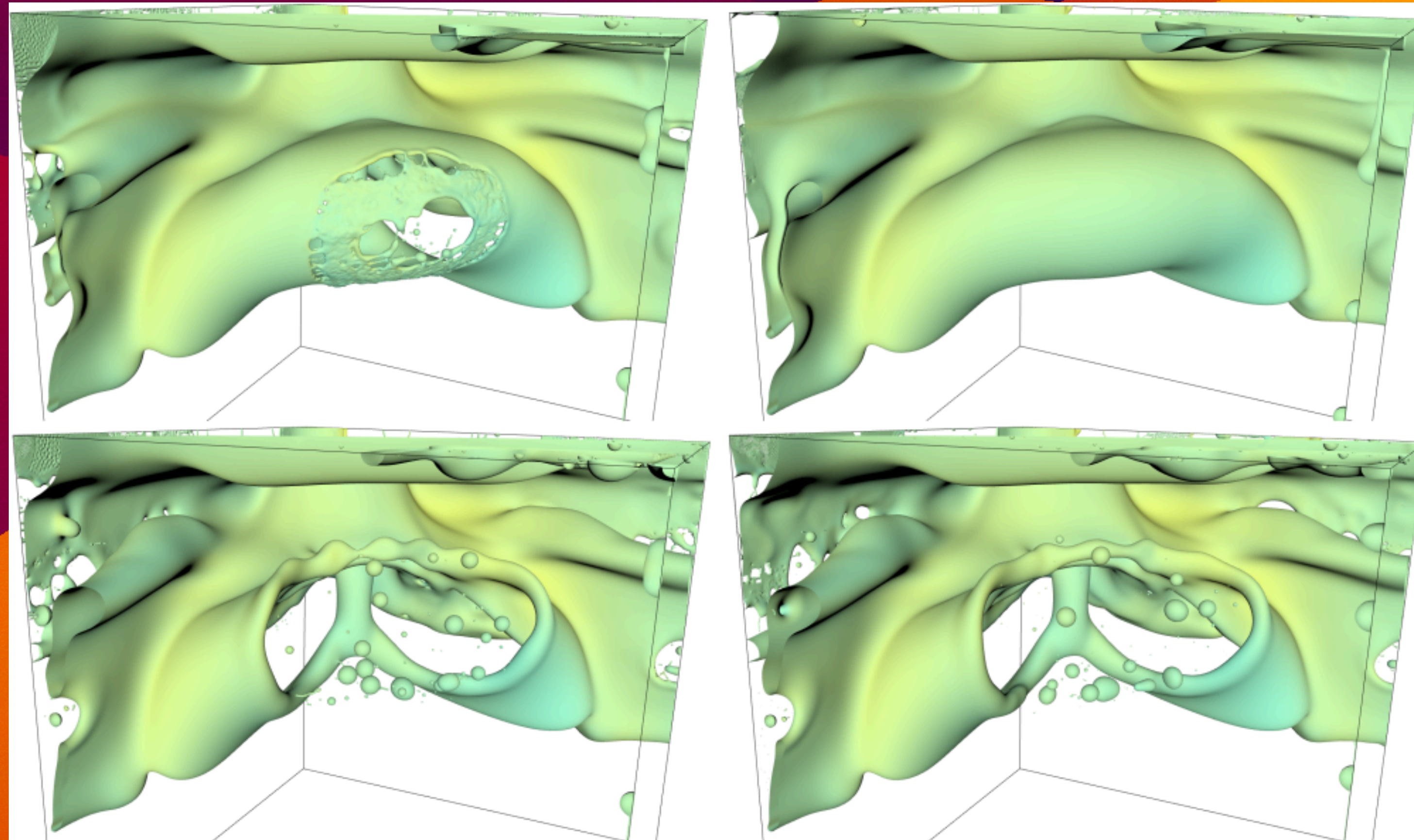
Reduced thermal gradient



Verifying Thermo-compositional convection

Tremplin et al. 2019

Test hypothesis using hydrodynamics code (Ex : Basilisk) with exorem profiles



Conclusion



- Exorem/Exoris robuste models for studying atmosphere and interior
- Non négligeable amount of work to link models still (At high T_{irr} and molar weight)
- Work on exorem side - Atmosphere dynamics (Thermo-compositional convection) - Effect on evolution models compared to clouds
- Work on exoris to include Neptunian equations of state
- Study of internal degeneracies after retrieval
- Improving physics all round